Math 102: First Exam

February 24, 2004, 8 am - 9:10 am

Read the directions to each problem carefully. Show all of your work. This exam has 6 questions. Write your answers in your blue book.

Problem 1: (10 points each) Find the family of antiderivatives for each of the following functions.

(a) $f(x) = \dfrac{1}{x\ln(x)}$, (b) $g(x) = xe^{-x}$, (c) $j(x) = \tan^3(x)\sec^3(x)$.

Problem 2: (10 points each) Evaluate the following definite integrals.

(a) $\displaystyle\int_1^2 \frac{x\ dx}{\sqrt{6x - x^2}}$, (b) $\displaystyle\int_1^3 \frac{2\cos^2\theta\ d\theta}{1 + \cos(2\theta)}$, (c) $\displaystyle\int_{-2}^{-1} \frac{3x + 6}{x^3 + 2x^2 + x}\ dx$.

Problem 3: (10 points) Explain how the rule for integration by parts can be found from the product rule and the Fundamental Theorem of Calculus.

Problem 4: (6 points each) Do the sequences below converge or diverge? If they converge, find the limit.

(a) The sequence $\{a_n\}_1^\infty$ where $a_n = \dfrac{7n^2 - 3}{14n^2 + 2n + 1}$.

(b) The sequence $\{b_n\}_1^\infty$ where $b_n = 2\sin\left(\dfrac{\pi n}{2}\right) \cdot \cos\left(\dfrac{\pi n}{2}\right)$.

Problem 5: (6 points each) Do the series below converge or diverge? If they converge, find the value of the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{\sqrt{9n^2 + 1}}$, (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \left(\frac{e}{3}\right)^n$.

Problem 6: (6 points) Show how an improper integral helps to see that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges. Draw the correct picture and explain.

Bonus Problem: (5 points) Consider the sequence $\{a_n\}_{n=1}^\infty$ where a_n is the nth digit in the decimal expansion for π. For example, since $\pi \approx 3.14159\ldots$, we know that $a_1 = 1$, $a_2 = 4$, $a_3 = 1$, $a_4 = 5$, etc. Does this sequence converge? If so, what is the limit?