Math 102: Second Exam

April 1, 2004,   8 am - 9:10 am

Read the directions to each problem carefully. Show all of your work.
This exam has 7 questions.

Problem 1.(2 points each) Without justifying your answer, give an example of an alternating series which

a) diverges, b) converges conditionally, c) converges absolutely.

Problem 2.(6 points) State the theorem which we call the 'Comparison Test'.

Problem 3.(15 points each) Determine convergence or divergence of the following series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+7}}$ b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\cdot(\ln n)^{3/2}}$

Problem 4. (15 points each) Determine the interval of convergence for the following power series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)}{n!}\, x^n,$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}(2x)^n,$

Problem 5.(8 points each) Find power series representations for the following functions and state the radius of convergence in each case.

a) $\displaystyle T(x) = \int_0^x \sin(t^3)dt,$ b) $\displaystyle J(x) = \frac{x^2}{(1-x)^2}$

Problem 6.(6 points) David tries to estimate $\sqrt{5}$ by using the Binomial series. He works as follows

$$\sqrt{5} = 2\sqrt{1+\frac{1}{4}} = 2\left[1+\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-n+1\right)}{n!}\left(\frac{1}{4}\right)^n\right]$$

$$\approx 2\left[1+\frac{1}{2}\cdot\frac{1}{4}-\frac{1}{8}\left(\frac{1}{4}\right)^2+\frac{3}{48}\cdot\left(\frac{1}{4}\right)^3\right] = \frac{1145}{512}.$$

Estimate the error in his approximation. (There is no need simplify your answer, a complicated-looking number is OK.) Be sure to say what you are doing.

Problem 7.(6 points) Give an example of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ such that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, but $\displaystyle\sum_{n=1}^{\infty} a_n^2$ diverges. Show that your answer works.