EXTRA PROBLEM FOR MATH 211, HOMEWORK # 4

Problem: Consider the initial value problem

$$y' = y^2, \quad y(0) = 1$$

(1) Solve the initial value problem analytically (that is, by hand). What is the interval of existence for this solution?

(2) Use the Picard iteration technique to solve the problem. Do at least 5 steps. Look for a pattern and conjecture what the nth approximation will look like. Discuss why you think that your sequence of functions will converge to the solution you got in the first part. (Brush up on your power series knowledge if you must.)

(3) Use MATLAB to produce a graph of the solution and several of your iterations on a direction field for the equation with window $-2 \le t \le 2$, $-1 \le y \le 3$.

(*Hint: If you find the computations for the Picard method too tedious, you could always write a MATLAB script to compute the successive approximations.*)