Project #2
The Spruce Budworm

Due: May 11, 2005 by 5pm.

The second project is a continuation of the first project. It involves the study of the 1978 paper *Qualitative Analysis of Insect Outbreak Systems: The Spruce Budworm and Forest* by Ludwig, Jones, and Holling, in which they proposed an ingenious model of the interaction of the insect, the trees, and the predatory birds. We will refer to this paper as [LJH]. The first project covered about the first half of [LJH]. In the second half they outline a three dimensional model for the forest/budworm system. In this project you are asked to explain and analyze that model.

The model The model describes the forest/budworm system using three variables. B is the population of budworms per acre, S is the area of leaves per acre, and E is a variable that measures the energy level of the forest. The rates of change of each is modeled using a logistic equation with modifications. The basic differential equations are equations (20), (21), and (22) in [LJH].

You have four tasks:

1. Read [LJH]. A copy is available online.
2. Describe the model contained in equations (20), (21), and (22). Do not repeat the derivation as found there. Instead start with the equations, and explain each term. Assume here that you are addressing a fellow student in Math 211. You should be able to do this in about two typewritten pages.
3. Carry out the analysis in Step 4 on p. 321 of [LJH]. This amounts to recreating Figures 3 and 4 using `pplane6`, and then explaining their import.

   This section of [LJH] is a clever way of determining some information about the equilibrium points of the three dimensional system without actually computing them. Computing them is very hard. It is probably not possible to do so analytically. However, the procedure used in [LJH] discloses enough information.
4. Compute solutions of the three dimensional system using `ode45`. You should do this for at least two values of T_E, the threshold energy. One where T_E is small, around 0.01, resulting in type of periodic behavior discussed in the first project. It features growth of the forest, followed by a budworm outbreak and the collapse of the forest. The second computation should have T_E relatively large, around $T_e = 0.2$, where the response is not periodic.

   In both cases you should present plots of the solution. As you know, there are a lot of ways to represent the solutions of a system of equations. Explore the possibilities and present us with the ones that best illustrate what is happening. In addition describe the results in both ecological and mathematical terms.

You will notice that in [LJH] computations are done in the case when $T_E = 0$.

Hints and explanation There are several items that require further explanation.

variable names We understand that if you were writing this paper for other purposes you might be tempted to change to a more pleasing set of variable and parameter names. However, the graders have to grade hundreds of papers, and it would be difficult for them to accommodate differing notation. Therefore, please use the variable names used in [LJH]. If you want to use additional variables and/or parameters to help in your explanation, that will be allowed.

One simplification we insist on is that you use $K_E = 1$ from the beginning, so that K_E does not even have to appear in your report. You will notice that K_E is used throughout [LJH], but where it counts they set $K_E = 1$. This one simplification makes the equations much easier to read and understand.

parameters You will need to use numerical values for the many parameters in the model when you start to compute. You can choose these values yourself as long as they are compatible with Table 1 on page 327 in[LJH]. The only parameter for which a value range is not given is the energy threshold T_E, and these values are provided earlier.

scaling The variables B, S, and E have very different orders of magnitude. The result is that if you try to plot them on the same graph, only the largest of them (B in this case) will appear to change at all. You will have to find a way around this, and the proper way is to scale the variables. The energy E is already normalized to range between 0 and 1. To scale S, you should compare it to the largest value of S that you can expect in terms of the model. This is the parameter K_S. (Why?) Hence introduce $s = S/K_S$ and plot s instead of S. You should do the same with B. The largest you can expect B to be is $K' K_S$. (Why?) Therefore, introduce $b = B/(K' K_S)$.

While the previous paragraph is primarily concerned with Task 4, you will also have scaling problems in Task 3. You can proceed with S unscaled, but you will discover that `pplane6` does not deal well with variables that have different orders of magnitude. A better way is to rewrite the differential equation for S in terms of s before entering it into `pplane6`.

isoclines and nullclines The term *isocline* as it appears in [LJH] is a synonym for nullcline.

editing figures To complete Task 3 you will want to edit some `pplane6` display figures. To make this possible, use the menu command **View**→**Figure Toolbar**. To prevent the accidental plotting of more solution curves, use the menu command **Options**→**Make the Display Window Inactive**. You should be warned that rescaling the figure after adding items like text or arrows will not move those added items.

using function M-files The computations in Task 4 are not difficult, but getting the resulting figures to look just the way you want them to will require you to repeat the computation several times. We suggest that you use a function M-file to organize this work. The method is described in the Manual in Chapters 4 and 8. You should especially look at the section entitled **Function M-File Drivers and Subfunctions** starting on page 107.

Since you will be exploring different ways to represent the solution, it would be nice if MATLAB could do several representations at once in different figures. In fact MATLAB can do that. If you ask nicely, your instructor might tell you how.

And don't forget to use the F5 key.

passing parameters Even if you fix on all of the parameters once and for all, you will still be changing T_E. Be sure to read the sections in Chapter 8 of the Manual on passing parameters to `ode45`. This discussion starts on page 114.