Math 211: First Exam

Spring 2005

This exam has a time limit of one hour and thirty minutes.
Read the directions to each problem carefully.
Show your work.

Problem 1: This question has five parts, each worth 3 points. No partial credit.

A: What is the order of the equation $y' \cdot y'' = \cos(y''')$?

B: Which of the following differential equations is linear?

$$(i) \quad \cos(x)y' + \sin(x)y - \tan(x) = 0, \quad \text{or } (ii) \quad 6 + xy^2 + x^2 + y' = 0.$$

C: Which of the following equations is autonomous?

$$(i) \quad \frac{dy}{dx} = \frac{\cos(y)}{y^2 + x^2}, \quad \text{or } (ii) \quad \frac{dy}{dt} = 6y^2 - \ln(\cos(y)).$$

D: Is the function $y = x\ln(x) - x + 10$ a solution to the equation $e^{y'} = x^2$?

E: Write an example of a differential equation which is not in normal form.

Problem 2: (15 points) Find a general solution to $y' + \frac{2}{x}y = 8x$ by the method of variation of parameters.

Problem 3: (15 points) Use an integrating factor to solve the initial value problem $y' = 6y - e^x,\ y(0) = 0$. What is the interval of existence of this particular solution?

Problem 4: This problem concerns the equation $y^2/x + (2y\ln(x) + 1) \cdot \frac{dy}{dx} = 0$.

A: (5 points) Rewrite this equation in differential form, and show that this is an exact equation.

B: (10 points) Find an implicit expression for the general solution to this differential equation.

Problem 5: David runs a lemonade stand. Every day he must spend all of his money on supplies. Fortunately, every day he takes in revenue equal to 2 dollars less than the square of the cash he had on hand before buying supplies.

A: (8 points) Write a differential equation which models the amount of money $M(t)$ dollars that David has t days into the enterprize.

B: (8 points) Use qualitative analysis to draw a phase line for this equation. Is the whole thing worth it if David started with 3 dollars?

Problem 6: This problem concerns the equation

$$x' = f(x,t) = t \cdot x^{2/3}.$$

A: (8 points) Are there any initial conditions which can not be satisfied by any solution to the equation? Either give an example or say how you know no such example exists.

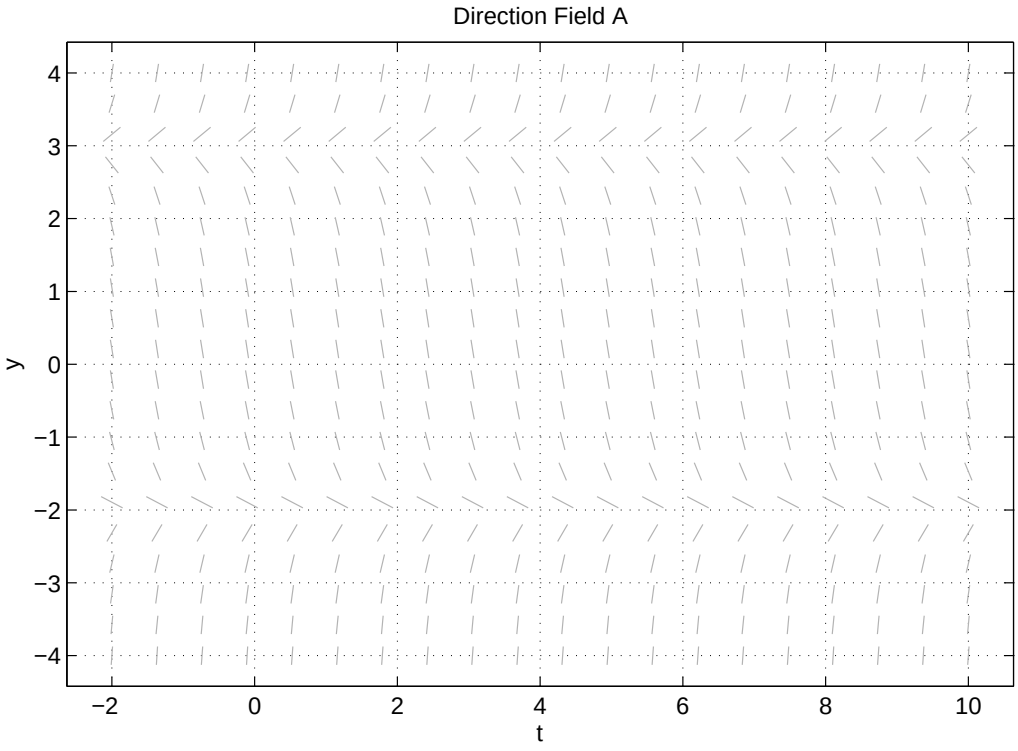
B: (8 points) Are there any initial conditions which are satisfied by more than one solution? Either give an example or say how you know no such example exists.

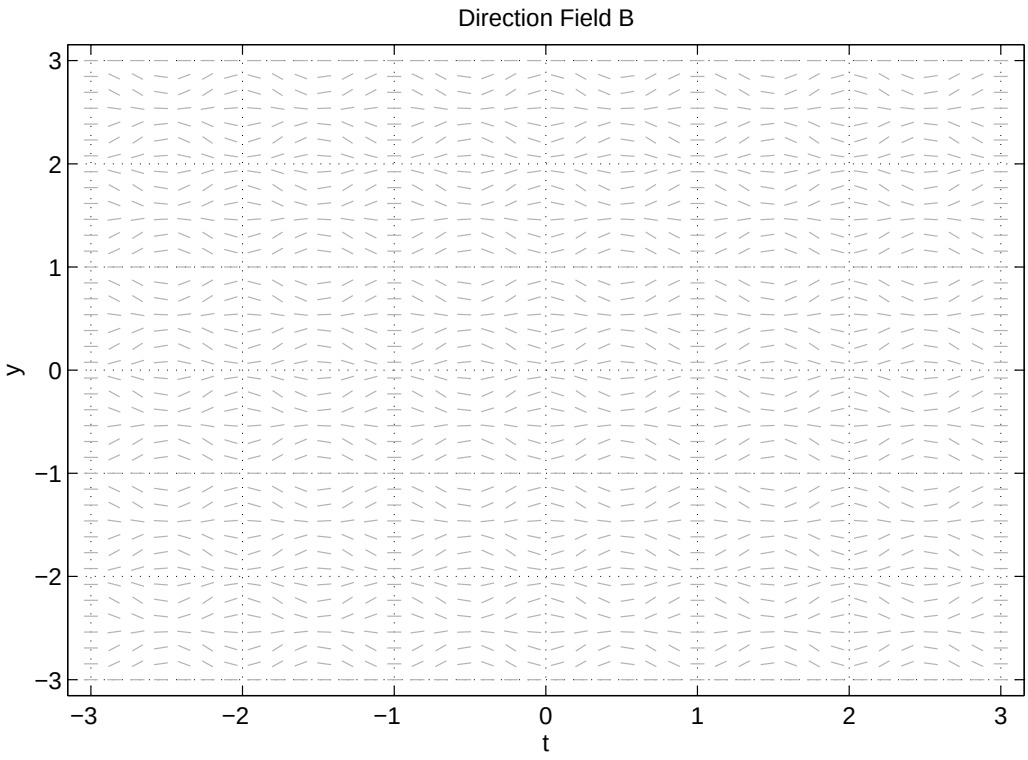
Problem 7: (8 points) Match the direction fields on this page and the next with the corresponding equation.

Equation One: $y' = y^2 - y - 6$

Equation Two: $y' = (y + t)^2$

Equation Three: $y' = \sin(2\pi y) \cdot \sin(2\pi t)$

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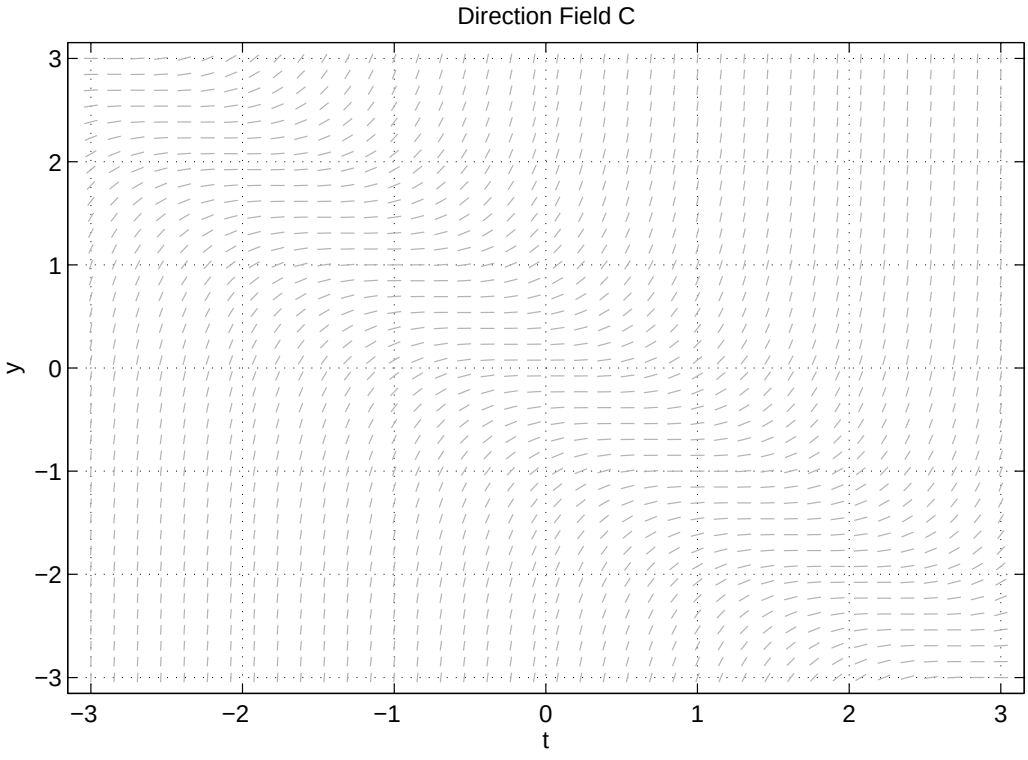
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