HOMEWORK # 1: MATH 211, SPRING 2005

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1. Exercises from the text

1.1. **Chapter 1.1.** In exercises 3 and 7 of Chapter 1.1, we are to model applications with a differential equation.

Problem 3 The maximum population is 100 ferrets, and we are told that the growth of the population of ferrets is proportional to the product of the population with the difference between the actual population and the maximum sustainable population.

So, let $P(t)$ denote the number of ferrets at time t. Then

$$\frac{dP}{dt} = C \cdot P \cdot (100 - P)$$

where $C > 0$ is some constant of proportionality. (By the way, this is a *logistic equation.*) For us, C is positive because we want the derivative dP/dt to be negative if $P > 100$. In this case there are too many ferrets!

Problem 7 The room temperature is 77 degrees Fahrenheit and the thermometer is much colder. So we expect that the rate of change of the temperature should be positive. We are told this rate of change is proportional to the difference between the room temperature and the temperature of the thermometer.

So, let $T(t)$ denote the temperature of the thermometer in degrees Fahrenheit at time t. Then

$$\frac{dT}{dt} = C \cdot (77 - T),$$

where $C > 0$ is a constant. Again, the constant is chosen this way to get the sign correct.

1.2. **Chapter 1.3.** This section deals with actually solving some of the simplest differential equations

Problem 13 We are to find the general solution to the differential equation, indicate the interval of existence for the solutions, and sketch at least six members of the family of solutions. Our equation is

$$r' = \frac{1}{u(1-u)}. \tag{1}$$

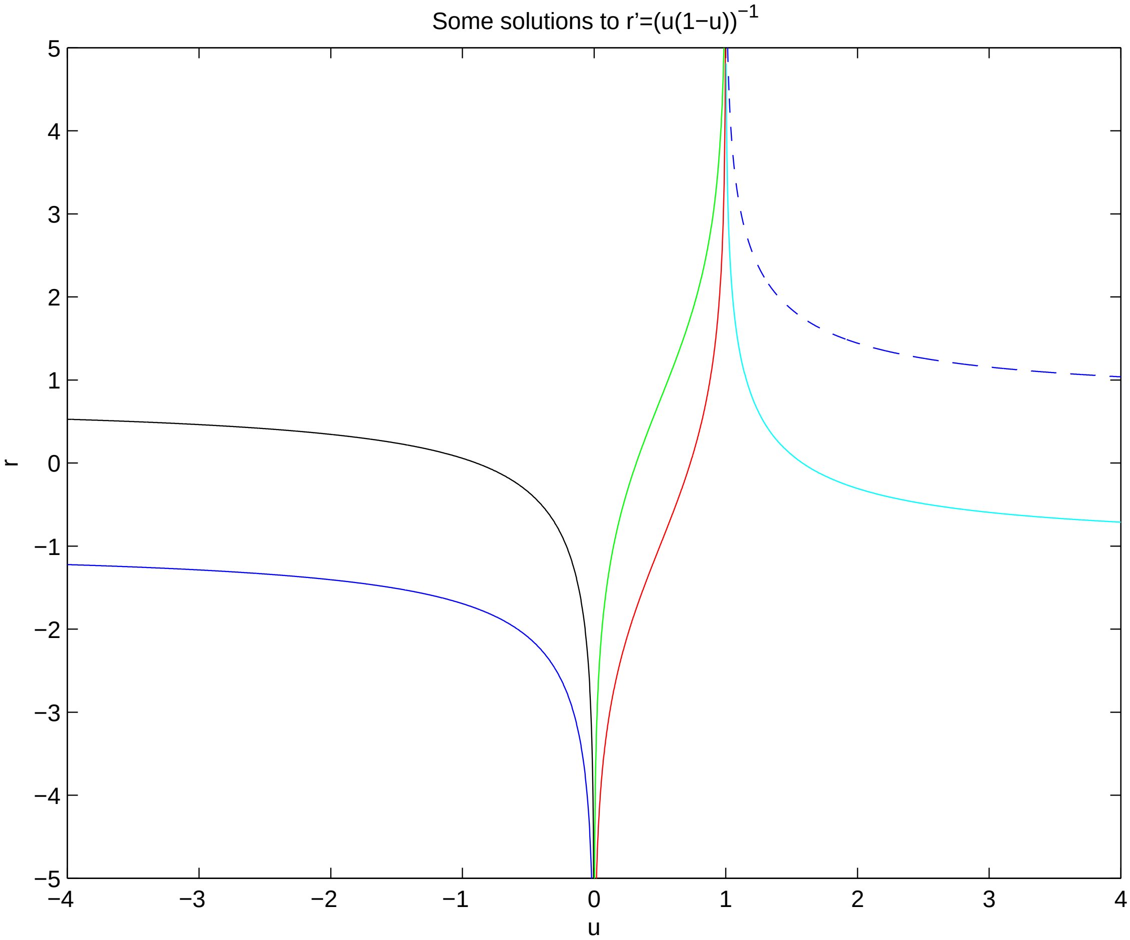
The right hand side of our equation is independent of r, so we have an equation we can solve by direct integration. We obtain by the method of partial fraction decomposition

$$r(u) = \int \frac{1}{u(1-u)}\, du = \int \left(\frac{1}{u} + \frac{1}{1-u} \right) du = \ln|u| - \ln|1-u| + C,$$

where C is an arbitrary constant of integration. This is the general solution to the differential equation 1.

Notice that this function is not defined at either $u = 1$ or $u = 0$. Thus the possible intervals of existence are $(-\infty, 0)$, $(0, 1)$ or $(1, \infty)$, depending on where you start.

Below is a graph of parts of solution curves corresponding to choices of $C = -1$ or 0.75 and choices of initial conditions which put us in each interval of existence for solutions. Each solution curve is in a different color. I made this plot with MATLAB, exported it as an encapsulated postscript file and put it into my document–you may want to try this at some point.



Problem 20 We are to find the solution to the initial value problem

$$P' = e^{-t}\cos(4t), \qquad P(0) = 1, \tag{2}$$

state the interval of existence, and sketch the solution.

Again, the differential equation is in normal form with right hand side independent of P, so we can find the general solution by integrating directly. This problem requires a trick to integrate which you should recall from second semester calculus. The idea is to integrate by parts twice, rearrange the result and pick out the solution. Or, you can quote your favorite table of integrals. In any case, the general solution to equation 2 has the form

$$P(t) = \int e^{-t}\cos(4t)\ dt = e^{-t}\left(\frac{4}{17}\sin(4t) - \frac{1}{17}\cos(4t)\right) + C,$$

where C is an arbitrary constant of integration. This is a case where it is important to double check by substituting into the original differential equation, as it is easy to make a small mistake in the calculation of two integration by parts steps.

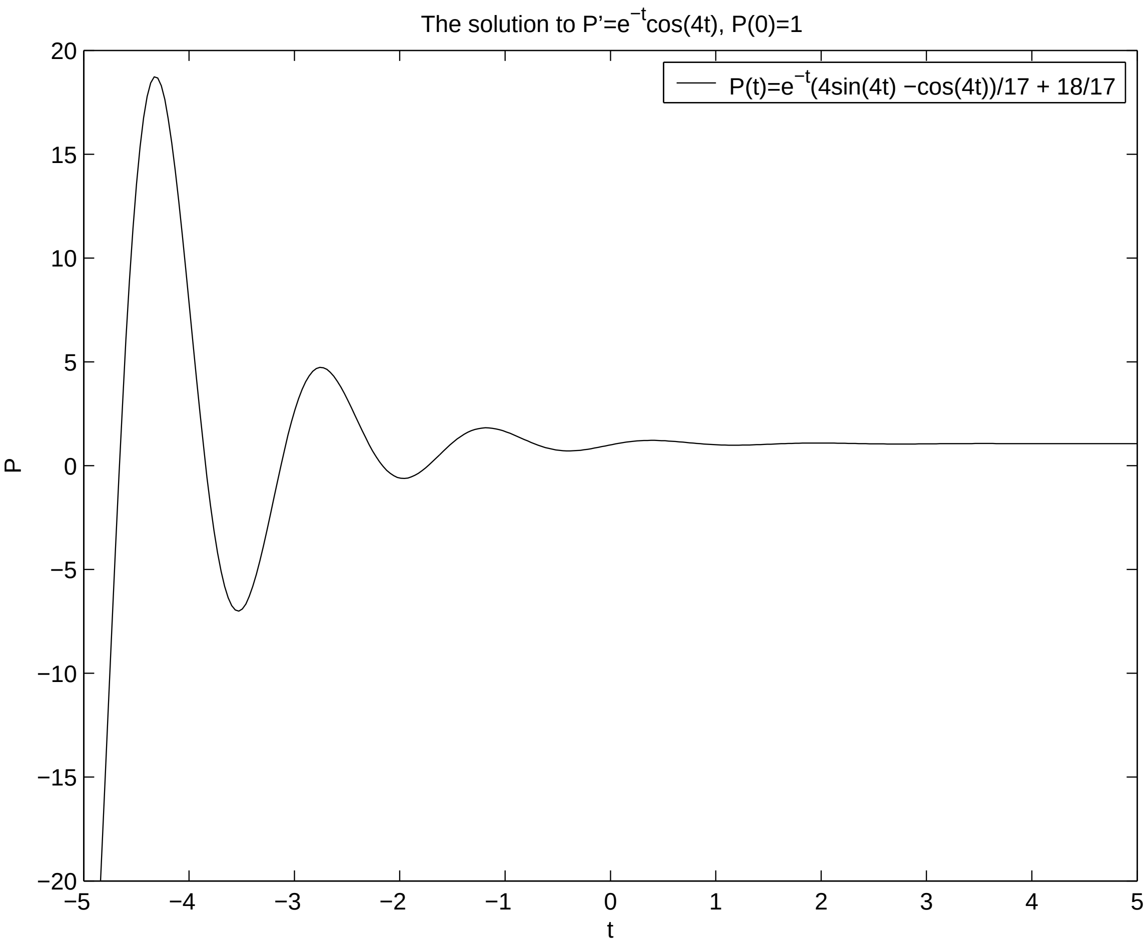
We are after that solution which also satisfies $P(0) = 1$. Using this information, we solve for C.

$$1 = P(0) = e^{-0}\left(\frac{4}{17}\sin(4\cdot 0) - \frac{1}{17}\cos(4\cdot 0)\right) + C = -\frac{1}{17} + C.$$

We thus find that $C = 1 + 1/17 = 18/17$, and our solution to the initial value problem 2 is

$$P(t) = e^{-t}\left(\frac{4}{17}\sin(4t) - \frac{1}{17}\cos(4t)\right) + \frac{18}{17}.$$

This solution clearly exists for all values of t, so our interval of existence is the whole line $(-\infty, \infty)$. We again use MATLAB to sketch a part of the solution curve.


The solution to P'=e^{−t}cos(4t), P(0)=1
P(t)=e^{−t}(4sin(4t) −cos(4t))/17 + 18/17
20
15
10
5
0
−5
−10
−15
−20
P
−5
−4
−3
−2
−1
0
1
2
3
4
5
t

M-files for my graphs

For textch1_3prob13.m

```
t=linspace(-4,0,1400);                %First we set up the intervals
s=linspace(0,1,400);                  % of existence
m=linspace(1,4,1200);
C=-1;                                 %The first choice of constant
y_1=log(abs(t)) - log(abs(1-t))+C;  %Our function, once for
y_2=log(abs(s)) - log(abs(1-s))+C;  % each interval
y_3=log(abs(m)) - log(abs(1-m))+C;
plot(t,y_1,s,y_2,'r',m,y_3,'c')     %We plot three solutions
hold on

C=0.75;                               %Change the constant
y_1=log(abs(t)) - log(abs(1-t))+C;  %Our function, once for
y_2=log(abs(s)) - log(abs(1-s))+C;  % each interval (again)
y_3=log(abs(m)) - log(abs(1-m))+C;

plot(t,y_1,'k',s,y_2,'g',m,y_3,'--')%Plot three more solutions
hold off axis([-4,4,-5,5])
xlabel('u')
ylabel('r')
title('Some solutions to r''=(u(1-u))^{-1}')
shg
```

Notice that in this file that I've separated the different parts of the graphs (because as solution curves they are not part of the same curve) by using three different intervals. This is a bit of an ad hoc solution, as I am still learning MATLAB, too.

For textch1_1prob20.m

```
t=linspace(-4,4,200);
y=exp(-t).*(17\4*sin(4*t)-17\cos(4*t))+18/17;
plot(t,y)
axis([-4,4,-5,5])
xlabel('t')
ylabel('P')
title('The solution to P''=e^{-t}cos(4t), P(0)=1')
legend('P(t)=(e^{-t}*(4\sin(4t)-cos(4t))+18)/17')
shg
```