

HOMEWORK # 7 SOLUTIONS

1. EXERCISES FROM THE TEXT CHAPTER 7

1.1. **Section 7.1. Problem 9.** We are working with 3×3 -matrices. The specific matrix A is not actually important. Inspecting the process for multiplying matrices, we see that the matrix we need is

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}.$$

This is related to what we discussed in class about elementary matrices, but we are multiplying on the right here where we multiplied on the left in the lecture. This corresponds to the fact that we are using column operations instead of row operations.

Problem 27. We are given vectors

$$\mathbf{x}_1 = \begin{pmatrix} 9 \\ 5 \end{pmatrix}, \quad \text{and} \quad \mathbf{x}_2 = \begin{pmatrix} -6 \\ -1 \end{pmatrix}.$$

The linear combination required is

$$2\mathbf{x}_1 + 3\mathbf{x}_2 = \begin{pmatrix} 0 \\ 7 \end{pmatrix}.$$

We write this linear combination as the matrix product

$$\begin{pmatrix} 9 & -6 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}.$$

This compares well with the above calculation.

Problem 37. We compute that

$$\begin{pmatrix} 10 & 0 & -1 \\ -5 & 8 & 3 \\ 3 & 6 & 6 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 \\ 11 \\ 9 \end{pmatrix}$$

Problem 44. Find the transpose of B .

$$B = \begin{pmatrix} -6 & -1 & 7 \\ -1 & 8 & -9 \end{pmatrix}, \quad B^T = \begin{pmatrix} -6 & -1 \\ -1 & 8 \\ 7 & -9 \end{pmatrix}$$

Problem 49. This system has matrix form as follows.

$$\begin{pmatrix} 3 & 4 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}.$$

Problem 52. This system has matrix form as follows.

$$\begin{pmatrix} 1 & 3 & -1 \\ -2 & 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}.$$

1.2. Section 7.2. Problem 2. This set is not a solution set to any system of linear equations in two variables. It is a disk about \mathbf{x}_0 of radius 2. It is easiest to see that it doesn't fit the requirement of being infinitely long. That is, it doesn't contain the lines through each pair of its points.

Problem 12. We are to find the parametric representation of the solution set to the equation

$$x_1 + 2x_2 - 2x_3 + x_4 = 2.$$

This is the set \mathcal{S} given by

$$\mathcal{S} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} +2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \mid s, t, u \in \mathbb{R} \right\}$$

This set is three dimensional.

Problem 13. We are to find a parametric representation of the solution set to the following system.

$$\begin{aligned} x_1 + 2x_2 - 2x_3 + x_4 &= 2 \\ x_2 - 3x_3 - x_4 &= 3. \end{aligned}$$

We eliminate x_2 from the first equation to obtain the equivalent system.

$$\begin{aligned}x_1 + 4x_3 + 3x_4 &= 8 \\x_2 - 3x_3 - x_4 &= 3.\end{aligned}$$

At this point, we can describe the solution set. It is two dimensional.

$$\mathcal{S} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -4 \\ 3 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 1 \\ 0 \\ 1 \end{pmatrix} \mid s, t \in \mathbb{R} \right\}$$

1.3. Section 7.3. In this section I'll omit all the tedious work, and just write out the answers.

Problem 10. $x = -4$, $y = -2$.

Problem 12. This system is inconsistent.

Problem 14. I'll write this one in a parametric description.

$$\mathcal{S} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13/40 \\ 1/4 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}.$$

Problem 16. Again, we use a parametric description. The solution is

$$\mathcal{S} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5/2 \\ 7/4 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1/4 \\ -5/8 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}.$$

1.4. Section 7.4. Problem 2. This system is inconsistent—the second row represents the equation $0x + 0y = 1$.

Problem 3. This system is also inconsistent. The third row is similar to the second row of the last problem.

Problem 9. This system has a solution set of

$$\mathcal{S} = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}.$$

Thus the homogeneous system $A\mathbf{x} = \mathbf{0}$ has many non-zero solutions, and the coefficient matrix A must be singular.

Problem 11. The only solution to this system is the zero vector. The coefficient matrix A is non-singular.

Problem 13. This matrix is singular simply because it represents more unknowns than equations. The solution set is

$$\mathcal{S} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \cdot \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}.$$

Problem 20. We already saw in problem 11 that this matrix is non-singular. We apply row reduction to find that

$$A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Problem 33. There are many ways to do this. One example is

$$\begin{aligned} x+2y &= 3 \\ x+2y &= 0. \end{aligned}$$

Problem 34. Again, there are many ways to do this. One example is

$$\begin{aligned} x+2y-4z &= 3 \\ x+2y-4z &= 0. \end{aligned}$$

1.5. Section Five.

Problem 4. The nullspace of this matrix is the zero subspace $\{\mathbf{0}\}$.

Problem 5. The nullspace of this matrix is the line

$$\mathcal{S} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \mid t \in \mathbb{R} \right\}.$$

Problem 10. The vectors $\mathbf{v}_1 = (-2, 3)^T$ and $\mathbf{v}_2 = (2, -6)^T$ are linearly independent because the nullspace of the associated matrix of columns $A = [\mathbf{v}_1, \mathbf{v}_2]$ is the trivial

subspace. This is easy to see once we sort out that the reduced row echelon form of A is the 2×2 identity matrix

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Problem 11. Again, we write out the associated column matrix and compute its reduced row echelon form. We obtain

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

This shows that equivalent equations to $t \cdot \mathbf{v}_1 + s \cdot \mathbf{v}_2 = 0$ are given by $t = 0, s = 0$. That is, the vectors are linearly independent.

Problem 12. Exactly as in the last problem, we can compute that the reduced row echelon form of the corresponding column matrix is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

Hence, these vectors are also linearly independent by the same reasoning.

2. EXERCISES FROM THE MANUAL CHAPTER 10

Problem 3. The MATLAB code required is as follows.

```
>> syms t y; y=10-(t^2)/2;
>> y*diff(y,t)+t*y
ans =
0
```

The final statement means that the given function is a solution to the given equation.

Problem 9. The independent variable here is x . The MATLAB code required for the other tasks is as follows.

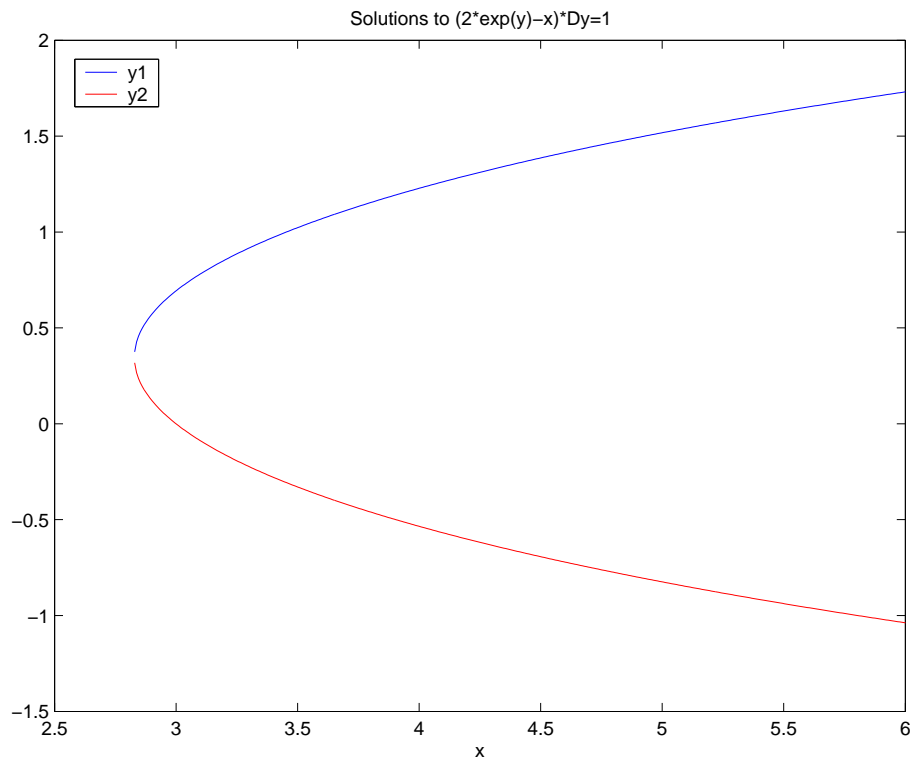
```
>> y=dsolve('(2*exp(y)-x)*Dy=1','x')
y =
[ log(1/2*x+1/2*(x^2-4*C1)^(1/2))]
[ log(1/2*x-1/2*(x^2-4*C1)^(1/2))]
>> C1=2; z=subs(y)
z =
```

```

[ log(1/2*x+1/2*(x^2-8)^(1/2))]
[ log(1/2*x-1/2*(x^2-8)^(1/2))]
>> g=z(1); h=z(2);
>> ezplot(g)
>> hold on
>> ezplot(h)
>> axis([2.5,6,-1.5,2])
>> legend('y1','y2')
>> title('Solutions to (2*exp(y)-x)*Dy=1')
>> shg

```

This produces the following picture of the two possible solutions we just plotted. I did a little hand-tweaking to get this picture.



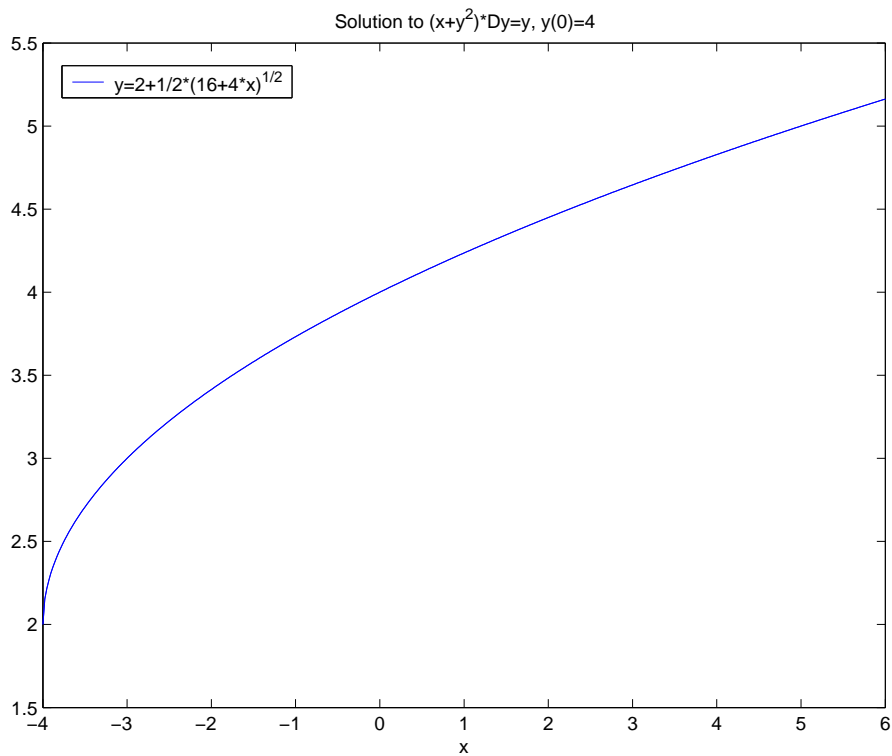
Problem 16. This problem is very similar to the last. Note that since we are given an initial condition, we can choose the proper function from the two that MATLAB returns to us. The independent variable is x , and we use the following code to execute the required tasks.

```

>> y=dsolve('(x+y^2)*Dy=y', 'y(0)=4', 'x')
y =
[ 2+1/2*(16+4*x)^(1/2)]
[      2-(x+4)^(1/2)]
>> g=y(1);
>> ezplot(g)
>> hold on
>> axis([-4,6,1.5,5.5])
>> legend('y=2+1/2*(16+4*x)^{1/2}')
>> title('Solution to (x+y^2)*Dy=y, y(0)=4')
>> hold off

```

This produces the following picture.



Note that along the way MATLAB gets a little confused and carries along the extra solution even though it doesn't fit the initial condition. Just dump it and move on.

Problem 27. This problem is a bit more challenging. The general form of solution to a logistic equation is

$$P(t) = \frac{a}{1 + be^{-ct}}.$$

So we execute the following MATLAB code.

```
>> clear
>> S=solve('50=a/(1+b)', '150=a/(1+b*exp(-c))', '250=a/(1+b*exp(-2*c))')
S =
    a: [1x1 sym]
    b: [1x1 sym]
    c: [1x1 sym]
>> S.a
ans =
300
>> S.b
ans =
5
>> S.c
ans =
log(5)
```

Notice that the command `solve` spits out a MATLAB array as its answer, and we must access that using the `S.a` type commands. The end result is that our function is given by

$$P(t) = \frac{300}{1 + 5e^{-\log(5)t}} = \frac{300}{1 + 5^{1-t}}.$$

This makes it clear that the limiting population is 300 and we can easily evaluate

$$P(3) \approx 288.4615.$$

Finally, we give the graph required.

