MATH 211 LECTURE 2

1. Solutions to Differential equations

We review the terminology for *first order differential equations*, *normal form*, and discuss the idea that a solution must be continuous and almost always continuously differentiable.

example $\cos(y' \cdot y \cdot t) = 1$, has particular solutions $y = 3/4$ and $y = \sqrt{4\pi \cdot \ln|t|}$. Mention that this is a strange equation because it is not in normal form (leads to two types of multiple-valuedness). Explain how this shows why we use the normal form for our study (i.e. for theorems).

2. Geometry of First Order ODE

We put our differential equation in normal form as often as we can (sometimes non-trivial). The differential equation $x' = f(x, t)$ is exactly a way of specifying the derivative of some unknown function $x = x(t)$ for each value of x and t. That is, given a current "time" and a current "value" for the unknown function, the diff eq tells us what the current derivative is. We then use the graphical interpretation of the derivative as the slope of the tangent line at a point! This leads to *direction fields* (or *slope fields*). Show how to use `dfield` to do it. Run through a couple of stock examples in the gallery.

3. Separable equations

We discuss how to do this in symbols by formal rearrangement on $y' = f(y)g(t)$.

example $y' = y$, $y(2) = 7$. Solution is $y = 7e^{x-2}$. Mention through this example how these often lead to *implicit solutions*. Discuss difference with *explicit solutions*.

Discuss the real justification for the method of separation of variables through change of variables/chain rule.

example $y' = \dfrac{2\pi}{ty}$, Solution is $y^2 = 4\pi \ln|t|$. Again, talk about implicit/explicit and loss of solutions. Also, use this equation to discuss *interval of existence*. This one has two: $(-\infty, -1)$ and $(1, \infty)$.

example (stuck at implicit solution) $y' = \dfrac{e^{-y}}{1+y}$, $y(0) = 2$. Solution is $ye^y = x + 2e^2$.

And I didn't actually get to this last one...

example (for students to follow all the way through) $y' = x^2y^2$, $y(0) = 1/2$. Solution is $y = \dfrac{1}{2 - x^3/3}$. Mention efficient use of initial conditions.