Math 211: Lecture One

Theron J Hitchman

January 13, 2005

Differential Equations

Begin with mention of course web page, first-day handout. Mention prerequisites, exams, homework, projects, reading of book and manual. take questions. ADVERTISE FOR VIGRE.

Things to cover: What is a differential equation?—How is it like a regular equation, how is it different? What do we mean by a solution? How do we check if we have one? Compare with an algebraic situation, say solving $x^2 + 6x + 8 = 0$ to get $x = -2, -4$. Start with $y' = y$.

natural questions: Is there a solution? If there is one, how many are there?

Use a simple separable equation ($y' = y$) with integration and introduce *general solution*, *particular solution* and *initial conditions*.

Why do we care? Modelling (hence word problems–elementary school wasn't for nothin') and rates of change. Give examples: (pick some easy ones and some cool ones). some terminology to introduce along the way: Order, systems.

Examples from Population Dynamics: Exponential growth/decay $y' = 12y, \quad y(0) = 100$, Logistic model $y' = 12y - 600y^2, \quad y(0) = 100$, predator-prey systems $R' = AR - BF, \quad F' = CR - DF$ (with some initial conditions).

Example from physics: a simple harmonic oscillator $x'' = 36\pi^2 x$. (something that makes 3 complete oscillations over a standard time unit.)

Example from Atmospheric dynamics: The Lorenz equations (1963)

$$\begin{aligned} x' &= A(y - x) \\ y' &= Rx - y - xz \\ z' &= xy - Bz \end{aligned}$$

(These describe the motion of an an idealized atmosphere of only one particle under "convection rolls".)

Mention the **problem**: In general, differential equations are nearly impossible to solve.

Ways to solve differential equations:

Dumb Luck Sometimes, the solution stares us in the face. examples are almost all simple linear equations of first order. (exponential equation, $x'' = x$)

Advanced Dumb Luck Through hard work and cleverness, you can perform some manipulation of your variables and show that your problem is the same as one you can already solve by the dumb luck method. Or at least you can find some solutions that way. example: integrating factors.

Numerically Admit the defeat of your pencil and paper approach and get a big computer to approximate your solution. This is relatively new as a mathematical technique (Math = thousands of years old, powerful computers= 40-50 years old). Problems: Errors and how to keep them small. (and be sure about it).

Give up... and try something else. The idea is to try to describe qualitatively, rather than quantitatively, the general nature of solutions. That is give up the numbers and look for fairly descriptive pictures. It's a bit like telling the police not that the guy who stole your wallet was TJ Hitchman, a math professor at Rice University, but instead a caucasian male, about 5'11", with light brown hair. You give up exact identification, or even identification up to some small error, and go for important distinguishing features.

Technically, there are other methods, but these sum up the big ones.

In our class, we will focus on all four of these. Parts one and two would have consumed the entirety of any diffeq class before 1900 (and a great many after). We still need to learn some of this stuff, though we'll spend a bit less time on option two than our great-great grandfathers did. This will require putting our calculus thinking caps on and using pen and paper.

In reality, the third and fourth methods are pretty well intertwined. We use computers to draw helpful pictures and pick out "approximate" solutions and divine as much as we can from this. In these cases we will use a powerful computer algebra system called MATLAB and some add-ons put together by Prof. Polking. This may seem like all we need these days, so why still do any of the first two?

Well, a real answer is still pretty nice to have when you can get it. Also, much of the analysis of the cases which force us into numerical and qualitative mode proceeds by trying to compare with a nice neat system that we can solve exactly. We'll get to this later. (this is "linearization")

A general plan for the semester: First order equations that we can do by dumb luck and some of the more useful tricks covered by Advanced dumb luck. We'll also discuss the main abstract theorem of differential equations, and get our first pass at the qualitative analytical approach. This is roughly chapters 1 and 2 of the text.

Next we shall discuss some aspects of modelling. This is not mathematics proper (by my snobbish definition), but really applied mathematics. This is the art of turning what we "know" scientifically into mathematics. It is one giant set of word problems, really. This is described in chapter 3.

By this point we'll have encountered some things which make it necessary to try numerical approximation. And that is our next topic. This comprises chapter 6 of the test. This is a lot harder than it sounds at first. It is more of an art.

With many of the major tools at our disposal in the first order case, we want to allow a bit more freedom in the equations. So we will encounter higher order equations and systems. We'll see how to make an equation simpler (in a couple of different ways) by making it into a system. But in order to handle systems of equations, we need linear algebra. This is the study of vectors, vector spaces, matrices and linear transformations. This is chapter 7.

Then, armed with new weapons, we attack systems with basically the same outline. We handle simple systems that we can solve (almost) by dumb luck, then we discuss how to adapt numerical methods to systems (pretty easy actually–the hard part is done), and then we'll look into qualitative methods and apply them to harder equations. This is the material of chapters 8-10. Along the way we will absorb the stuff on second order equations in chapter 4, because we'll know how to write them as systems in 2 (or sometimes 3) dimensions.

With what time we have left:

1 The easiest types of equations to solve

Concerned with first order differential equations. These methods work for those put into "normal form"

$$x' = f(x, t).$$

This type of notation can be overly general to the point of uselessness, but after a while, it can help organize thinking.

If we happen to have an equation in the form $x' = f(t)$ where the function on the right doesn't depend on x, then we can solve by integrating! That is, using the fundamental theorem of calculus:

$$x(t) = \int f(t)\ dt.$$

This depends on being able to integrate the function–it's own bag of tricks. In practice, this gives a general solution, as the antiderivative on the right is defined only up to an additive constant. To get a particular solution we need more information. We need an initial value problem!

$$x' = f(t), \qquad x(t_0) = x_0$$

The solution to this problem should be

$$x(t) = \int_{t_0}^{t} f(t)\ dt + x_0$$

Examples: $y' = \frac{1}{t}, y(0) = 3$, and $y' = e^{7t}, y(0) = 3$.