211 LECTURE 19

Systems of Differential Equations, part II

A note on something I should have stressed last time. A system needs an initial condition, too. But for a system, an initial condition has more information in it because it is a vector equation. In the case of reducing an nth order equation to an nth order system, we see that we really need initial values of the function and its first (n-1) derivatives to solve an initial value problem. That is, we need n pieces of information.

1. Qualitative analysis and long-term behavior

Systems can be solved exactly even less often than single differential equations, so qualitative analysis becomes even more important.

Equilibrium points are important, but we get more stuff to keep track of because of the extra dimensions. We also need *nullclines.* These come in a variety of types, one for each equation in the system, and we need to keep them separate. It can be useful to have different colors on the plot or use dotted and dashed lines. Discuss direction of travel across nullclines, how nullclines intersect to give equilibrium points (be careful about the dimensions).

Consider some examples from basic systems and learn to draw the pictures.

Example (Lotka-Volterra)

$$\begin{aligned} F' &= -2F + 3RF \\ R' &= 2R - 4RF \end{aligned}$$

Example(Lorenz's equations)

$$\begin{aligned} x' &= -3x + 3y \\ y' &= x - y - xz \\ z' &= -2z + xy \end{aligned}$$

Get Matlab to draw the Lorenz example and have it ready.

2. Linear systems

We need to discuss several general terms which are more or less familiar in this new setting. These include *linear system, forcing term, vector notation, coefficients, homogeneous, inhomogeneous.*

We then need to recall the better version of the existence theorem for linear equations and how it applies here (i.e. nearly verbatim).

Finally, we discuss the structure of the solution set to a homogeneous linear system. Note that any linear combination of solutions (with constant coefficients) is also a solution. Discuss linear independence of *functions.* Note that we can check this for homogeneous systems at <u>one point</u>, by the uniqueness theorem. So we are interested in finding a basis set of functions, or a fundamental set of solutions. That is, we need to find n linearly independent solutions to describe the solution set to the n dimensional linear homogeneous system. (Note that the expected number of "different" solutions is the same as the number of constants needed to specify a solution in the existence/uniqueness theorem.) Discuss using the determinant to check linear independence, and the terminology of the *Wronskian.*

Examples

$$\begin{aligned} x' &= x + 2y \\ y' &= 2x + y \end{aligned}$$

Has solutions of $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e^{3t} \\ e^{3t} \end{pmatrix}$. In fact, these are linearly independent.

$$\begin{aligned} x' &= -2x + 3z \\ z' &= 2x - 4z \end{aligned}$$

Has solutions of $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2e^{-t} \\ 2e^{-t} \end{pmatrix}$. These are not linearly independent.