Math 211 Lecture 4 Examples

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Mixing Problem # 1

A tank initially contains 100 gallons of water in which is dissolved 2 lbs of salt. A salt-water solution which contains 1lb of salt for every 4 gallons of solution enters the tank at a rate of 5 gallons per minute. Solution leaves the tank at the same rate allowing for a constant volume in the tank.

a Use an analytic method to determine the eventual salt content in the tank.

b Use a numerical solver to do the same and compare.

Setup of solution

Well, $Vol = 100\ gal$ is constant.
$x(t) =$ the amount of salt (in lbs) in the tank t minutes after we begin is not.
$x(0) = 2$.

$$\text{Rate in} = \left(\frac{1\ lb}{4\ gal}\right)(5\ gal/m)$$

$$\text{Rate out} = \left(\frac{x(t)\ lb}{100\ gal}\right)(5\ gal/m)$$

So we get $\dfrac{dx}{dt} = \dfrac{5}{4} - \dfrac{x(t)}{20}, \quad x(0) = 2.$
We care about the limit $\lim_{t\to\infty} x(t)$. Let's solve this with an integrating factor to stay sharp.

Mixing Problem # 2

A tank contains 500 gallons of salt-water solution containing 0.05 lbs of salt per gallon of water. Pure water is poured into the tank and a drain at the bottom of the tank is adjusted so as to keep the volume of the solution constant. At what rate should the water be poured into the tank to lower the salt concentration to 0.01 lb/gal of water in under one hour?

Setup of solution

Again, $vol = 500\ gal$ is constant.

$x(t) =$ the amount of salt in pounds t minutes into the process.

$x(0)/500 = 0.05\ lb/gal$, so $x(0) = 25\ lbs$.

let A be the rate in gal/min that we want.

Rate in $= (0 lb/gal)(A\ gal/min)$

Rate out $= \dfrac{x(t) lb}{500 gal}(A\ gal/min)$

So $\dfrac{dx}{dt} = -\dfrac{Ax}{500}, \quad x(0) = 25.$

We want to choose A so that $\frac{x(60)}{500} \leq 0.01\ lbs/gal$, that is so that $x(60) \leq 5\ lbs$.