MATH 211 LECTURE 9

Mathematical Modelling, part I

1. The idea of modelling

The aim is to assess some phenomenon in the real world and translate it into mathematics. The real world is messy, mathematics is more often crisp and clear.

Basically, one must first make a set of hypotheses and work out the details from these. These hypotheses should represent in sentences what one knows about a particular system. The hard part is to do the translation. How does the way the world works get put into mathematical language? We'll do an example or two to see how it can be done.

2. the limits of modelling

Yes, the math gives exact numbers. But these predictions should be taken with a grain of salt. The model must be reinterpreted when applying it back to the real world.

At best, the model is an idealization of your situation, or an approximation. One must be willing to abandon the model and start over if it makes too many incorrect predictions.

3. Some simple models of population growth

Some of the easiest models to get at are basic population grwoth models. We study the two most important and most widely used here.

3.1. Malthusian/exponential growth/decay.

Hypotheses:

- A population of unicellular organisms, *nothing else.*
- "unlimited resources"
- The probability that a cell divides during the time interval $(t, t + \Delta t)$ is equal to $b\Delta t$.
- The probability that a cell dies during the time interval $(t, t + \Delta t)$ is equal to $d\Delta t$.

Derive the equation, rename the parameter $r = b - d$ the 'reproductive rate'. The standard exponential growth equation is $P' = aP$.

We then discuss the solution and interpretation. Note that the unchecked growth of the population is a bit "unphysical".

3.2. **the logistic model.** Hypotheses:

- A population of unicellular organisms, *nothing else.*
- "limited resources" introduce competition.
- The probability that a cell divides during the time interval $(t, t + \Delta t)$ is equal to $(a + bP(t))\Delta t$.
- The probability that a cell dies during the time interval $(t, t + \Delta t)$ is equal to $(c + dP(t))\Delta t$.

Derive the equation, rename the parameter $r = a - c$ the 'natural reproductive rate', and the parameter $K = r/(b + d)$ the 'carrying capacity'. The logistic model is $P' = r(1 - \frac{P}{K})P$.

Discuss solution, behavior of solutions. Do a phase line analysis and look at solutions on a direction field. Discuss the nature of the choices made in the hypothesis: arbitrary, but somehow the *simplest* which make sense to describe what we want. This is the art behind modelling.