Math 211, Spring 2005
Exam II

Write your answers in your blue book. Show all of your work. Write the pledge and sign your name on the front of the book.

Problem 1 (20 points) Describe the numerical process known as *Euler's method* as it applies to a single equation. Be sure to draw the relevant picture and describe the algorithm correctly. What can one say about the error that this method gives?

Problem 2 (5 points) Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

Compute the matrix $A \cdot B^T$.

Problem 3 (5 points) Find the nullspace of the matrix

$$C = \begin{pmatrix} 2 & 3 \\ \pi & 3\pi/2 \end{pmatrix}.$$

Express it as a parameterized set.

Problem 4 (10 points) Compute the determinant of

$$D = \begin{pmatrix} 1 & 3 & 5 \\ 7 & 4 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

by using row reduction.
Compute the determinant of

$$E = \begin{pmatrix} 6 & 5 & 0 & 3 \\ 0 & 0 & 2 & 1 \\ 5 & 5 & 1 & 3 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

by expanding along a row or column.

Problem 5 (10 points) Is it possible that a system of n equations in n unknowns be inconsistent but have matrix form $A\mathbf{x} = \mathbf{b}$ where $\det(A) \neq 0$? If it is possible, give an example. If it is not possible, explain why not.

Problem 6 (10 points) Is it possible to have a system of three equations in two unknowns which is inconsistent but has matrix form $A\mathbf{x} = \mathbf{b}$ where A has trivial nullspace? If it is possible, give an example. If it is not possible, explain why not.

Problem 7 (15 points) Find the set of solutions to the following system of linear equations. If the set is more than one vector, express it as a parameterized set.

$$\begin{aligned} 4x+2y-6z &= 2 \\ x\ -y+3z &= 0 \\ 5x-2y+6z &= 1 \end{aligned}$$

Problem 8 (10 points) Translate the following linear third order equation into an equivalent first order system. Write the resulting system in matrix-vector form as compactly as you can.

$$x''' = \cos(t)x'' - \sin(t)x' + e^t x + \tan(t)$$

Problem 9 (15 points) Use qualitative analysis techniques to draw a rough picture of the phase plane for the system below. Your picture should include equilibrium points, nullclines and the general direction of travel in the regions between nullclines. Be sure that your graph is clearly labelled.

$$\begin{aligned} x' &= 2x-y-x^3 \\ y' &= x \end{aligned}$$