# Math 381: First Exam

September 26, 2003

Read the directions to each problem carefully. Show your work when necessary. This exam has 8 questions on 5 pages and lasts 50 minutes.

Name:________________________________________

Pledge:

**Problem 1:**(6 points) Identify the order of the following partial differential equations:

a) $\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial z}{\partial x} + 4z^2 \dfrac{\partial z}{\partial y} = 0$

b) $\dfrac{\partial^7 z}{\partial x \partial y^6} \dfrac{\partial^{14} z}{\partial x^{14}} = -1.$

c) $\left(\dfrac{\partial z}{\partial x} - y\right)\left(\dfrac{\partial z}{\partial y} - x\right) = 0.$

**Problem 2:**(6 points) Show the work necessary to verify that for arbitrary $C^2$ functions $f$ and $g$, the expression

$$y = f(x + at) - g(x - at)$$

is a solution of the partial differential equation

$$\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial x^2} = 0,$$

where $a$ is a positive constant.

**Problem 3:**(6 points) Write a partial differential equation of first order which is quasi-linear but not linear.

**Problem 4:**(15 points) Find the general solution of the following partial differential equation in a form solved for $z$.

$$\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = e^{-z}$$

**Problem 5:** Consider the partial differential equation

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0 \tag{1}$$

a) (6 points) Find equations which describe the characteristic curves of this equation.

b) (3 points) Show that the positive $x$-axis, $\sigma(t) = (t, 0), t \geq 0$, is free for the initial condition $f(t) = t^2$.

c) (6 points) Find the integral surface of equation (1) which contains the curve $y = 0$, $z = x^2$. Draw a picture of this surface, or describe it well in a sentence.

d) (2 points) How many solutions to equation (1) are there which satisfy the initial conditions that $z = 25$ when $x^2 + y^2 = 5$?

**Problem 6:**(20 points) Find the Fourier coefficients of the square wave function

$$w(x) = \begin{cases} 1, & 0 \le x \le \pi \\ -1, & -\pi < x < 0 \end{cases},$$

and write down the first four terms of the Fourier series.

**Problem 7:**(10 points) State the *all-important* corollary to Riemann's theorem. (Sometimes we call this the Riemann-Lebesgue Lemma.)

**Problem 8:**(20 points) In a paragraph or two, discuss the nature of convergence of a Fourier series of a function which is continuous except for a finite number of finite size jumps. Be sure to cite examples and state results clearly. You need not prove anything.