Homework Assignment # 5
Due Friday, October 31, 2003.

The first bunch of problems each have two parts: Part (a) Find the Legendre series expansion for the function indicated. Part (b) Construct a graph of the first three interesting partial sums of the Legendre series on the same set of axes as the original function. (Each problem is worth 7 points: 5 for the series, 2 for the graph.)

1. $f(x) = x^2$ for $-1 \leq x \leq 1$.
2. $g(x) = x^3$ for $-1 \leq x \leq 1$.
3. $h(x) = x^4$ for $-1 \leq x \leq 1$.
4. $j(x) = x^5$ for $-1 \leq x \leq 1$.
5. $l(x) = |x|$ for $-1 \leq x \leq 1$.

Problem 6: (10 points) Let $f(x) = \cos(2\pi x)$ for $-1 \leq x \leq 1$. Find the first five non-zero terms of the Legendre series for f. Graph these in a way that shows the convergence of the partial sums of the Legendre series to f.

Problem 7: (10 points) How does your answer for problem 6 compare to the Taylor series for $\cos(2\pi x)$ at $x = 0$? How do the partial sums of order 10 compare? Both methods are approximations by polynomials, so which one is "better"?

Bonus Problem 1: (10 points) Prove the least squares property for Legendre polynomials:

If f and f^2 are integrable over $(-1, 1)$, and $s_n(x)$ denotes the n-th partial sum of the Legendre series for f, then

$$\int_{-1}^{1} (f(x) - s_n(x))^2 \, dx \leq \int_{-1}^{1} (f(x) - p(x))^2 \, dx$$

for any polynomial $p(x)$ of degree less than or equal to n.

(Hint: Follow the proof for Fourier series.)