Energy Yield and Visual Impact
Studies of the Berlin Wind Project

Samuel M. Arons

A thesis submitted in partial fulfillment
of the requirements for the
Degree of Bachelor of Arts with Honors
in Physics

WILLIAMS COLLEGE
Williamstown, Massachusetts

12 May 2004
Acknowledgments

I would like to acknowledge the many people without whose guidance, patience, and company I would not have been able to successfully complete this work. First, I would like to thank my ‘official’ advisor Prof. Sarah Bolton (Physics) and my ‘second’—but equally important—advisor Prof. David Dethier (Geosciences) for their incredible support and advice throughout these past eleven months. I am also indebted to Prof. Dwight Whitaker, my ‘third’ advisor, who helped me greatly in understanding and thinking about fluid mechanics and air flow; to Prof. Jeff Strait for taking the time to read and comment on a draft; and to the other members of the physics department for their support over the course of the year. I would like to thank Prof. Karen Kwitter and Dr. Steven Souza of the Astronomy department for their help in the initial stages of the visual impact study, as well as Prof. Enrique Peacock-Lopez (Chemistry), the resident Mathematica expert, for spending an afternoon with me puzzling over some incomprehensible error messages.

I would not have had the opportunity to work on this project without all those who came before me. I owe gratitude to Reed Zars ’77 for having the crazy idea of a Williams College wind farm in the first place, to Thomas Black ’81 for his perseverance in making WWERP a reality, and to Nicholas Hiza ’02, Fred Hines ’02, and Chris Warshaw ’02 for rediscovering the project, dusting it off, and handing part of it to me for safekeeping for a few months. I also thank the Center for Environmental Studies and the Thomas Black fund for supporting my work during the summer of 2003. I am indebted to Dr. Paul Bieringer of MIT’s Lincoln Laboratory for providing me with wind data and coaching me along in the initial stages of analysis.

I could not have accomplished the mundane—but crucial—details of day-to-day work without the help of the following people: Larry Mattison, George Walther, Emile Ouelette, Bryce Babcock, Sharron Macklin, Heather Main, Joe Moran, other behind-the-scenes members of B&G, Jody Psoter, Carol Marks, Barb Swanson, Sandy Brown, Sarah Gardner, Hank Art, Sandy Zepka, Walt Congdon, Martha Staskus, Andrew Gillette, Hayley Horowitz ’04, Zach Yeskel ’04, Emily Gustafson ’04, and any others that I may have inadvertently left out.

Finally, I would like to thank Ellie Frazier ’05 for putting up with and comforting a sometimes overworked and cranky person. I owe deepest thanks to my parents, without whom none of this would have been possible.
Abstract

The Berlin Wind Project is a Williams College-sponsored study of the potential for electricity generation by a 7–9-turbine wind farm at Berlin Pass (Berlin, NY). Two questions that must be addressed in assessing the project’s viability are: (1) How much energy could the proposed wind farm produce in a year? and (2) What would be the turbines’ visual impact? In this thesis, I present both the answers to these questions and the techniques necessary to obtain them.

I first conclude that AWS Truewind’s wind resource maps predict energy yield with an accuracy of approximately 16 ± 14% in the northern Berkshire/Taconic region, and that the maps also predict directional distributions quite reasonably. I next conclude that a 7-turbine wind farm at Berlin Pass could produce 35 ± 8 million kW-hr per year, or 163 ± 21% of Williams College’s 2002–2003 energy use on average. Because of natural fluctuations in wind speed, this value could vary by as much as an additional ±10% from year to year. Furthermore, since the prevailing winds at the Pass blow from the WNW and the ridgeline runs NNE–SSW, turbine shading should not cause substantial energy losses—though there would likely be some losses from a moderate SSW wind component. Assuming a net turbine cost (sale price + installation) of $1.24 million ($8.65 million for 7 turbines) and an average wholesale electricity price of $38/MW-hr, the farm could pay for itself in, very roughly, 6.5 ± 1.6 years.

In addition, based on the results of the visual impact study—some 59 potential views of the wind farm from various locations within a 20 km radius of the Pass—I conclude that the turbines are likely to be visible from quite a few locations throughout the region. However, from a number of these locations the turbines may appear to be quite small and could remain unnoticed by all but the most careful observers.

In light of these results, my recommendation to the College is to continue researching the Project while maintaining an open dialog with the local communities.
Advisor:
Professor Sarah R. Bolton, Physics

Copyright © 2004 Samuel Max Arons
# Contents

Acknowledgments i

Abstract ii

List of Figures vii

List of Tables xii

1 Introduction 1
   1.1 Format of the Thesis ........................................... 2
   1.2 Site Location .................................................. 3
   1.3 Project History ............................................... 3

2 Fluid Flow in Simple Geometries 6
   2.1 The Navier-Stokes Equation .................................. 6
   2.2 Flow Between Parallel Planes ................................. 7
   2.3 Flow Through a Tube .......................................... 9
   2.4 The Need for Wind Data ..................................... 10

3 Turbines and Prediction Methodology 11
   3.1 Modern Wind Turbines ....................................... 11
      3.1.1 Turbine Properties .................................... 11
      3.1.2 Power Curves and Mechanical Efficiency .......... 13
   3.2 Energy Production ........................................... 16
      3.2.1 Speed Distributions .................................... 16
      3.2.2 Truewind ............................................... 19
      3.2.3 Speed Extrapolation .................................. 20
      3.2.4 Air Density ........................................... 21
      3.2.5 Error Analysis ......................................... 23

4 Brodie Data Analysis 24
   4.1 Energy Production Estimates ................................. 24
      4.1.1 Log Law .................................................. 26
      4.1.2 Power Law, $\alpha = 1/7$ ............................. 32
      4.1.3 Power Law, Variable $\alpha$ ........................... 34
      4.1.4 Weibull Distribution .................................. 37
CONTENTS

4.1.5 Rayleigh Distribution ........................................... 42
4.1.6 The Local Wind Resource ...................................... 44
4.2 Comparison of the Six Energy Estimation Methods .......... 45
  4.2.1 Truewind Accuracy ........................................... 48
4.3 Comparison of 1997 to 1998 Log Law Predictions ............... 49
4.4 Wind Direction at Brodie Mountain ............................ 51
4.5 Summary of Results ............................................. 53

5 Lincoln Labs Data Analysis ........................................ 55
  5.1 Energy Production Estimates .................................. 55
    5.1.1 Log Law: MTR ............................................. 57
    5.1.2 Weibull Distribution: MTR ............................... 59
    5.1.3 Truewind Accuracy ....................................... 61
    5.1.4 Taconic Ridge (TCN) & Notch Road (NCH) .............. 63
  5.2 Wind Direction at Mt. Rainer .................................. 64
  5.3 Summary of Results ............................................. 66

6 Black Data Analysis ................................................ 69
  6.1 Black’s Instruments and the In Situ Divisor ................. 69
  6.2 Energy Production Estimates .................................. 71
    6.2.1 Log Law .................................................. 72
    6.2.2 Weibull Distribution .................................... 74
    6.2.3 Truewind Accuracy ...................................... 76
  6.3 Wind Direction at Berlin Pass ................................ 77
  6.4 Summary of Results ............................................. 81

7 Energy Yield at Berlin Pass ....................................... 82
  7.1 Truewind’s Accuracy and the $\gamma$ Factor .................. 82
  7.2 Energy Yield at Berlin Pass .................................. 84
    7.2.1 Monthly and Annual Figures ............................ 84
    7.2.2 Error Analysis ......................................... 86
    7.2.3 Comparison with Black’s Thesis Data Prediction ...... 87
  7.3 Wind Direction at Berlin Pass ................................ 88
  7.4 Summary of Results ............................................. 89

8 A Met Tower on the Roof ......................................... 90
  8.1 Equipment List & Initial Testing ............................. 90
  8.2 The MSL Roof Met Tower ...................................... 92
    8.2.1 Installation .............................................. 92
    8.2.2 Preliminary Results .................................... 93

9 Visual Impact ...................................................... 97
  9.1 Viewshed & Image Collection .................................. 97
## CONTENTS

9.2  The Digital Camera .......................... 100
   9.2.1 Angular Pixel Size: Theoretical ............. 100
   9.2.2 Angular Pixel Size: Experimental ............ 102
9.3  Creating the Images ........................ 103
9.4  Conclusions ................................ 104

10 Conclusions & Further Research .......... 106
   10.1 Conclusions ................................ 106
   10.2 Future Research ............................ 107

A  The Betz Limit and Power Curves .......... 109
   A.1 Available Wind Power ....................... 109
   A.2 Betz Limit .................................. 110
   A.3 Power Curves ............................... 112

B  WindData.gs Program Code and Sample Output 113
   B.1 WindData.gs Code ............................ 113
   B.2 Sample Command-Line Output ............... 118
   B.3 Sample Output File ......................... 120

C  The Global Wind Resource .................. 121
   C.1 Solar Radiation & Terrestrial Absorption ...... 121
   C.2 A Giant Heat Engine in the Sky .............. 121
   C.3 Extracting Aeolian Energy .................. 122
   C.4 United States Energy Consumption .......... 122
   C.5 World Energy Consumption ................. 123
   C.6 Conclusions ............................... 124

D  Met Tower Diagrams ....................... 126

E  Visual Impact Images ....................... 131

Bibliography ................................ 193
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The proposed site of the Berlin Wind Project at Berlin Pass.</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>Flow between two parallel planes.</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>The solution to the Navier-Stokes equation for the ‘jet stream’ between</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>two parallel planes.</td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>Flow through a circular cylinder.</td>
<td>9</td>
</tr>
<tr>
<td>3.1</td>
<td>Four General Electric 1.5 MW turbines in Gatun, Spain.</td>
<td>12</td>
</tr>
<tr>
<td>3.2</td>
<td>Rime ice shedding from a turbine.</td>
<td>13</td>
</tr>
<tr>
<td>3.3</td>
<td>Theoretical maximum power curve compared to a real GE 1.5 MW power curve.</td>
<td>14</td>
</tr>
<tr>
<td>3.4</td>
<td>A close-up view of the GE 1.5 MW power curve.</td>
<td>15</td>
</tr>
<tr>
<td>3.5</td>
<td>The efficiency of GE’s 1.5 MW turbine.</td>
<td>15</td>
</tr>
<tr>
<td>3.6</td>
<td>The distribution of wind speeds at Brodie Mountain in January 1998.</td>
<td>17</td>
</tr>
<tr>
<td>3.7</td>
<td>Two Weibull distributions with different $k$ values.</td>
<td>18</td>
</tr>
<tr>
<td>3.8</td>
<td>Two Rayleigh distributions with different $\bar{U}$ values.</td>
<td>19</td>
</tr>
<tr>
<td>3.9</td>
<td>Comparison of log and power laws.</td>
<td>21</td>
</tr>
<tr>
<td>4.1</td>
<td>The location of Brodie Mountain with respect to Berlin Pass.</td>
<td>25</td>
</tr>
<tr>
<td>4.2</td>
<td>Monthly log law production estimates for Brodie Mountain, 1998.</td>
<td>27</td>
</tr>
<tr>
<td>4.4</td>
<td>Monthly energy demand at Williams College, July 2002–June 2003.</td>
<td>29</td>
</tr>
<tr>
<td>4.5</td>
<td>Monthly fixed-$\alpha$ power law production estimates for Brodie Mountain,</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>1998.</td>
<td></td>
</tr>
<tr>
<td>4.6</td>
<td>Monthly variable-$\alpha$ power law production estimates for Brodie Mountain,</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>1998.</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>Monthly Weibull distribution production estimates for Brodie Mountain,</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>1998.</td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>Close-up of the monthly Weibull distribution estimates.</td>
<td>39</td>
</tr>
<tr>
<td>4.9</td>
<td>Monthly Rayleigh distribution production estimates for Brodie Mountain,</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>1998.</td>
<td></td>
</tr>
<tr>
<td>4.10</td>
<td>The monthly wind resource at Brodie Mountain, 1998.</td>
<td>46</td>
</tr>
<tr>
<td>4.11</td>
<td>Predicted energy yield vs. production method, by month. Brodie Mountain,</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>1998.</td>
<td></td>
</tr>
</tbody>
</table>
4.13 Predicted annual energy yield vs. production method. Brodie Mountain, 1998. .................................................. 47
4.14 Comparison of 1997 and 1998 log-law energy predictions for Brodie Mountain. .................................................. 50
4.15 Empirical wind rose diagram for Brodie Mountain, 1998. .................................................. 51
4.16 Truewind-predicted wind rose diagram for Brodie Mountain. .................................................. 52

5.1 Map of the Berkshire Mesonet. .................................................. 56
5.2 Monthly log law production estimates at Mt. Rainer, 2001. .................................................. 58
5.3 Monthly Weibull distribution production estimates at Mt. Rainer, 2001. .................................................. 60
5.4 Comparison of annual energy production estimates for Mt. Rainer, 2001. .................................................. 62
5.5 Comparison of annual energy production estimates for the Taconic Ridge, 2001. .................................................. 64
5.6 Empirical wind rose diagram for Mt. Rainer, June–November 2001. .................................................. 65
5.7 Truewind-predicted wind rose diagram for Mt. Rainer. .................................................. 67

6.1 The approximate locations of Black’s met towers. .................................................. 70
6.2 Monthly log law production estimates for Berlin Pass, 1980–81. .................................................. 73
6.3 Monthly Weibull distribution production estimates for Berlin Pass, 1980–81. .................................................. 75
6.4 Comparison of annual energy production estimates for Berlin Pass. .................................................. 77
6.5 Empirical wind rose diagram for Berlin Pass, 1980–81. These data are likely inaccurate. .................................................. 79
6.6 Truewind-predicted wind rose diagram for Berlin Pass, 1980–81. .................................................. 80

7.1 Comparison of energy yield at Berlin Pass predicted by Black’s thesis data and by Truewind’s maps. .................................................. 87
7.2 Truewind-predicted wind rose diagram for Berlin Pass. .................................................. 88

8.1 The approximate location of the MSL met tower on the campus of Williams College. .................................................. 91
8.2 Installation of the met tower on the MSL roof. .................................................. 93
8.3 Wind speed distribution at the roof of the Morley Science Laboratory. .................................................. 94
8.4 Hourly speed speed averages at the roof of the Morley Science Laboratory. .................................................. 95
8.5 Empirical wind rose diagram for the roof of the Morley Science Laboratory. .................................................. 96

9.1 The viewshed of the BWP’s 7 turbines. .................................................. 98
9.2 The 45 viewpoints of the visual impact study. .................................................. 99
9.3 Schematic diagram of the angular size of a turbine. .................................................. 100
9.4 Diagram of a camera’s lens. .................................................. 101
9.5 Experimental setup to determine angular pixel size. .................................................. 103
9.6 The three steps for placing the turbines in the images. .................................................. 105

A.1 Air of density $\rho$ flows through a cylinder of area $A$ at speed $U$. .................................................. 109
A.2 An actuator disc and stream tube. .................................................. 110
D.1 A schematic diagram of the MSL roof met tower. .......................... 127
D.2 A technical drawing of the mast holder seen from above. ................. 128
D.3 A technical drawing of the mast holder in vertical cross section. ........... 129
D.4 A technical drawing of the flange rings and mounting arms. ............... 130

E.1 A map of the 45 viewpoints for which potential views of the proposed wind farm were generated. ................................................... 132
E.2 The 45 viewpoints for which visual impact images were created. ........... 133
E.3 {Viewpoint #1} View from the summit of Pine Cobble, Williamstown, MA. Distance to site: 10.1 km. ........................................ 134
E.4 {Viewpoint #2} View from Pine Cobble development, Williamstown, MA. Distance to site: 9.1 km. ........................................ 135
E.5 {Viewpoint #3} View from the intersection of Cole Avenue and North Hoosac, Williamstown, MA. Distance to site: 8.4 km. ................. 136
E.6 {Viewpoint #3} View from the intersection of Cole Avenue and North Hoosac, Williamstown, MA (zoomed in). Distance to site: 8.4 km. .... 137
E.7 {Viewpoint #4} View from Whitman Road, Williamstown, MA. Distance to site: 6.9 km. Turbines obscured by vegetation (possibly all year 'round).138
E.8 {Viewpoint #5} View from outside Thomson Chapel, Williams College campus, Williamstown, MA. Distance to site: 7.1 km. Turbines obscured by vegetation (in summer). ............................... 139
E.9 {Viewpoint #6} View from the Taconic Golf Course, Williamstown, MA. Distance to site: 7.0 km. Turbines obscured by vegetation (possibly all year 'round). ............................... 140
E.10 {Viewpoint #7} View from Stone Hill, Williamstown, MA. Distance to site: 5.8 km. Turbines obscured by vegetation (in summer). ............ 141
E.11 {Viewpoint #8} View from the Mt. Greylock High School football field, Williamstown, MA. Distance to site: 5.9 km. ....................... 142
E.12 {Viewpoint #8} View from the Mt. Greylock High School football field, Williamstown, MA (zoomed in). Distance to site: 5.9 km. ............ 143
E.13 {Viewpoint #9} View of Berlin Pass from Five Corners, Williamstown, MA. Distance to site: 6.2 km. Turbines obscured by vegetation (in summer).144
E.14 {Viewpoint #10} View from Stony Ledge, Williamstown, MA. Distance to site: 10.6 km. Turbines obscured by vegetation (in summer). .... 145
E.15 {Viewpoint #11} View from the summit of Mt. Greylock, Adams, MA. Distance to site: 12.8 km. ........................................ 146
E.16 {Viewpoint #11} View from the summit of Mt. Greylock, Adams, MA (zoomed in). Distance to site: 12.8 km. ............................... 147
E.17 {Viewpoint #12} View from the top of the Greylock War Memorial, Adams, MA. Distance to site: 12.8 km. ............................... 148
E.18 {Viewpoint #12} View from the top of the Greylock War Memorial, Adams, MA (zoomed in). Distance to site: 12.8 km. ............................... 149
E.19 {Viewpoint #13} View from Mt. Prospect where the AT takes a 90° turn, Williamstown, MA. Distance to site: 10.2 km. ....................... 150
E.20 {Viewpoint #14} View from Blair Road, Williamstown, MA. Distance to site: 8.4 km. .................................................. 151
E.21 {Viewpoint #15} View from Pattison Road, Williamstown, MA. Distance to site: 9.6 km. .................................................. 152
E.22 {Viewpoint #15} View from Pattison Road, Williamstown, MA (zoomed in). Distance to site: 9.6 km. .................................................. 153
E.23 {Viewpoint #16} View from Route 2 at Luce Road, Williamstown, MA. Distance to site: 8.8 km. .................................................. 154
E.24 {Viewpoint #16} View from Route 2 at Luce Road, Williamstown, MA (zoomed in). Distance to site: 8.8 km. .................................................. 155
E.25 {Viewpoint #17} View from the Stop ‘n’ Shop parking lot, North Adams, MA. Distance to site: 10.2 km. .................................................. 156
E.26 {Viewpoint #17} View from the Stop ‘n’ Shop parking lot, North Adams, MA (zoomed in). Distance to site: 10.2 km. .................................................. 157
E.27 {Viewpoint #18} View from Massachusetts Avenue, Blackinton, MA. Distance to site: 10.7 km. .................................................. 158
E.28 {Viewpoint #19} View from the Protection Avenue bridge, North Adams, MA. Distance to site: 11.4 km. .................................................. 159
E.29 {Viewpoint #20} View from Route 2, 1 km from North Adams, MA. Distance to site: 13.4 km. Turbines obscured by vegetation (in summer). 160
E.30 {Viewpoint #21} View the MassMoCA parking lot, North Adams, MA. Distance to site: 14.4 km. Turbines obscured by buildings. ................. 161
E.31 {Viewpoint #22} View from the top of the hairpin turn, North Adams, MA. Distance to site: 18.3 km. .................................................. 162
E.32 {Viewpoint #23} View from the old Williams College ski area parking lot, Williamstown, MA (zoomed in). Distance to site: 1.1 km. ........... 163
E.33 {Viewpoint #24} View looking south from the proposed wind farm site at Berlin Pass, Berlin, NY. Distance to site: n/a. .............................. 164
E.34 {Viewpoint #25} View from the summit of Berlin Mountain, Berlin, NY. Distance to site: 1.8 km. Turbines obscured by vegetation (in summer) . 165
E.35 {Viewpoint #25} View from the summit of Berlin Mountain, Berlin, NY (zoomed in). Distance to site: 1.8 km. Turbines obscured by vegetation (in summer) . .................................................. 166
E.36 {Viewpoint #26} View from 400 m northwest of the intersection of Green Hollow Road and Cold Spring Road, Berlin, NY. Distance to site: 3.8 km. 167
E.37 {Viewpoint #26} View from 400 m northwest of the intersection of Green Hollow Road and Cold Spring Road, Berlin, NY (zoomed in). Distance to site: 3.8 km. .................................................. 168
E.38 {Viewpoint #27} View from 500 m west of the intersection of Green Hollow Road and Cold Spring Road, Berlin, NY. Distance to site: 4.2 km. 169
E.39 {Viewpoint #28} View from the intersection of Route 22 & Elm Street (Green Hollow Road), Berlin, NY. Distance to site: 7.0 km. Turbines obscured by vegetation (possibly all year 'round). .......................... 170
E.40 {Viewpoint #29} View from Route 22, 1.3 km south of Berlin, NY. Distance to site: 7.0 km. .................................................. 171
E.41 {Viewpoint #29} View from Route 22, 1.3 km south of Berlin, NY (zoomed in). Distance to site: 7.0 km. ......................... 172
E.42 {Viewpoint #30} View from Route 22, 200 m north of Satterlee Hollow Road, Berlin, NY. Distance to site: 7.3 km. .............. 173
E.43 {Viewpoint #31} View from Route 40, 1.3 km west of Berlin, NY. Distance to site: 8.4 km. ........................................... 174
E.44 {Viewpoint #31} View from Route 40, 1.3 km west of Berlin, NY (zoomed in). Distance to site: 8.4 km. .......................... 175
E.45 {Viewpoint #32} View from Cherry Plain State Park, Stephentown, NY. Distance to site: 14.1 km. Turbines obscured by topography and vegetation. 176
E.46 {Viewpoint #33} View from Miller Road, Berlin, NY. Distance to site: 12.5 km. Turbines obscured by topography and vegetation. ....... 177
E.47 {Viewpoint #34} View from Dutch Church Road, Berlin, NY. Distance to site: 12.0 km. Turbines obscured by topography and vegetation. ... 178
E.48 {Viewpoint #35} View from Grafton Lakes State Park, Grafton, NY. Distance to site: 15.6 km. Turbines obscured by topography and vegetation. 179
E.49 {Viewpoint #36} View from 100 m north of the intersection of Routes 2 & 22, Petersburg, NY. Distance to site: 6.7 km. .................. 180
E.50 {Viewpoint #37} View from 350 m west of the intersection of Routes 2 & 22, Petersburg, NY. Distance to site: 7.0 km. ................. 181
E.51 {Viewpoint #37} View from 350 m west of the intersection of Routes 2 & 22, Petersburg, NY (zoomed in). Distance to site: 7.0 km. .... 182
E.52 {Viewpoint #38} View from Route 2 near the border of Petersburg and Grafton, NY. Distance to site: 9.3 km. Turbines obscured by topography and vegetation. .......................... 183
E.53 {Viewpoint #39} View from Route 22, 2.8 km north of Petersburg, NY. Distance to site: 8.8 km. ................................. 184
E.54 {Viewpoint #39} View from Route 22, 2.8 km north of Petersburg, NY (zoomed in). Distance to site: 8.8 km. ...................... 185
E.55 {Viewpoint #40} View from Route 2 at East Hollow Road, Petersburg, NY. Distance to site: 3.7 km. ............................. 186
E.56 {Viewpoint #41} View from Route 2, 800 m east of East Hollow Road, Petersburg, NY. Distance to site: 4.2 km. .................. 187
E.57 {Viewpoint #41} View from Route 2, 800 m east of East Hollow Road, Petersburg, NY (zoomed in). Distance to site: 4.2 km. .... 188
E.58 {Viewpoint #42} View from the Taconic Crest Trail, 3 km north of where Route 2 crosses Petersburg Pass, just west of the New York/Vermont border. Distance to site: 4.8 km. ............................... 189
E.59 {Viewpoint #43} View from Post Road, Pownal, VT. Distance to site: 7.7 km. Turbines obscured by topography. .................... 190
E.60 {Viewpoint #44} View from Route 7, 3.5 km north of the Massachusetts border, Pownal, VT. Distance to site: 8.7 km. Turbines obscured by topography. ........................................... 191
E.61 {Viewpoint #45} View from top of Mann Hill Road, Pownal, VT. Distance to site: 9.8 km. Turbines obscured by topography. ........ 192
## List of Tables

4.1 Average monthly air temperatures, pressures, and densities for Brodie Mountain, 1998. ................................................................. 26
4.2 Log law energy estimates for Brodie Mountain, 1998. .................. 27
4.3 Fixed-α power law energy estimates for Brodie Mountain, 1998. .... 32
4.4 Variable-α power law energy estimates for Brodie Mountain, 1998. ... 36
4.5 Weibull distribution energy estimates for Brodie Mountain, 1998. ... 38
4.6 Values of c and k at several possible locations of the Brodie anemometer tower. ................................................................. 40
4.7 Rayleigh distribution energy estimates for Brodie Mountain, 1998. .... 43
4.8 Values of \( \bar{U} \) at several possible locations of the Brodie anemometer tower. 44
4.9 The wind resource at Brodie Mountain in 1998. .......................... 45
4.10 Monthly percentage differences between log law and Truewind at Brodie Mountain, 1998. ................................................................. 49
4.11 Monthly percentage differences between 1997 and 1998 log-law predictions for Brodie Mountain. ................................................................. 50
4.12 Summary of predicted annual energy yields for Brodie Mountain, 1998. 53

5.1 Site characteristics of the Berkshire Mesonet weather towers. ........ 56
5.2 Log law energy estimates for Mt. Rainer, 2001. ............................ 57
5.3 Seasonal c and k values for Mt. Rainer. ........................................ 59
5.4 Weibull distribution energy estimates for Mt. Rainer, 2001. .......... 60
5.5 Monthly percentage differences between log law and Truewind for Mt. Rainer, 2001. ................................................................. 62
5.6 Summary of predicted annual energy yields for Mt. Rainer, 2001. .... 66

6.1 Sensor calibration results for Black’s PASS100 anemometer. ........... 72
6.2 Log law energy estimates for Berlin Pass, 1980–81. ...................... 73
6.3 Seasonal c and k values for Berlin Pass. ...................................... 74
6.4 Weibull distribution energy estimates for Berlin Pass, 1980–81. ....... 75
6.5 Monthly percentage differences between log law and Truewind for Berlin Pass, 1980–81. ................................................................. 76
6.6 Summary of predicted annual energy yields for Berlin Pass, 1980–81. 81

7.1 Comparison of Truewind’s ‘overestimate percentage’ for four data sets. . 83
7.2 Annual c and k values for each of the 7 proposed turbines at Berlin Pass. 85
7.3  Annual Truewind predictions of energy yield for 7 turbines at Berlin Pass.  85
7.4  Comparison of energy predictions made from Black’s thesis data and Truewind’s maps. 87
7.5  Predicted annual energy yield and payback time for a 7-turbine wind farm at Berlin Pass. 89
9.1  Characteristics of the Olympus D-490 digital camera. 102
Chapter 1

Introduction

The purpose of this thesis is to address the question of wind power in the northwest Berkshire/northern Taconic region, focusing specifically on a ridge known as Berlin Pass. The Berlin Wind Project (BWP), a Williams College-sponsored study of the potential for electricity generation by a 7–9-turbine wind farm at the ridge, is particularly interested in the Pass because of the high wind speeds predicted to be prevalent there. Among the many issues the project faces in assessing the viability of the proposed wind farm are two of critical importance: (1) energy yield and (2) visual impact. The work presented herein, carried out between June 2003 and May 2004, represents an attempt to resolve these issues.

The question of potential energy yield is particularly acute for the BWP. Though in general, power output at a site is estimated using wind data from a year-long anemometer study, the project has so far been unable to obtain a permit from the town of Berlin, NY to erect such a tower. Furthermore, past studies of the project have included only rough estimates of annual energy production ([25], [2], [11]). Thus, if we seek a more reliable estimate—before anemometer data become available—we will have to obtain it in some other manner.

This thesis explores three distinct means of predicting energy yield in the absence of site-specific wind data. (1) To begin, we consider the possibility of analytically calculating the wind regime using the equations of fluid mechanics. It soon becomes apparent, however, that beyond the simplest models this method proves quite difficult. (2) Next, we analyze wind data from three nearby, ‘surrogate’ sites in the hopes that their wind regimes are similar enough to that at the Pass to reasonably approximate the energy production there. (3) At the same time, we predict energy yield both at these surrogate sites and at the Pass itself using Weibull coefficients from wind maps developed by AWS Truewind, LLC, an energy technology and atmospheric modeling firm.

Separately, the latter two methods can give only rough estimates of production at Berlin Pass—but together they conspire to predict energy yield much more accurately. That is, if we compare the so-called ‘Truewind’ predictions to the actual wind-data predictions at the surrogate sites, we can determine by what factor Truewind generally over- or underestimates energy yield. Armed with this knowledge, we can then use Truewind to estimate production at the Pass—and finally multiply by the empirical ‘over- (or under-) estimation factor’ to ‘re-scale’ the Truewind estimate and obtain the
1.1. FORMAT OF THE THESIS

The thesis is arranged in the following manner:

This chapter provides a brief history of the Berlin Wind Project and describes the proposed site at Berlin Pass.

Chapter 2 derives fluid flow in a few simple geometries from basic fluid mechanics.

1Personal communication with Nicholas Hiza, 10 May 2004.
Chapter 3 examines the modern wind turbine and derives several different methodologies for estimating energy yield, in addition to describing how to evaluate the uncertainty in those estimates.

Chapter 4 offers and compares six separate estimates of energy yield and wind direction at Brodie Mountain, using wind data collected by the Renewable Energy Research Laboratory (RERL) at UMass Amherst.

Chapter 5 presents an estimate of energy yield and wind direction at Mt. Raimer, using data collected by researchers at MIT’s Lincoln Laboratory.

Chapter 6 offers a preliminary estimate of energy yield and wind direction at Berlin Pass, using data collected by Thomas Black ’81.

Chapter 7 combines the results of Chapters 4–6 in order to rigorously predict the expected energy yield of a 7-turbine wind farm at Berlin Pass.

Chapter 8 describes tests performed on wind instruments mounted to a meteorological tower on the roof of Williams College’s Morley Science Center and presents the preliminary results of those tests.

Chapter 9 explains how the visual impact study of the Berlin Wind Project was realized. The visual impact images are presented in Appendix E.

Chapter 10 offers suggestions for future work that could follow from the results presented in the thesis.

1.2 Site Location

The site of the proposed wind farm is located at Berlin Pass, a ridge in the northern Taconic range joining Mt. Raimer to the north and Berlin Mountain to the south. The parcel of land, which is owned by Williams College and which is the former location of the College ski area, lies approximately 5.6 km (3.5 mi) to the west of Williamstown, MA, 6.4 km (4 mi) east of Berlin, NY, and less than 1.6 km (1 mi) south of where Route 2 crosses Petersburg Pass (Figure 1.1). The elevation of the site is approximately 670 m (2,220 ft) above sea level. Because the Pass represents a low point between Mt. Raimer and Berlin Mountain, the prevailing northwest winds are channeled through the site, making it an attractive location for electricity generation (after [11]).

1.3 Project History

What is now known as the Berlin Wind Project was first conceived in 1976 by Reed Zars ’77. During his senior year, Zars conducted an independent study to evaluate the feasibility of installing a small-scale wind farm at Berlin Pass. In a report to the Trustees entitled ‘The Proposed Wind Energy System for Williams College’, he concluded that if the college were to invest some $524,000 (1977 dollars) in three 200-kW machines, the
Figure 1.1: The proposed site of the Berlin Wind Project at Berlin Pass is outlined in red. The translucent irregular polygon is the property owned by Williams College.
farm would generate approximately $72,000 in electricity bill savings each year ([25], [2]).

Zars’ report, however, was admittedly preliminary, and he closed with an appeal for further research. For several years no additional progress was made until, in June of 1980, Thomas Black ’81 began work on his senior environmental studies thesis. Over the course of twelve months—from August 1980 to July 1981—Black collected wind data at the Pass as part of what he dubbed the ‘Williams Wind Energy Research Project’ (WWERP). These data, though unfortunately incomplete because of a vandalism problem at the site, allowed Black to conduct a more careful evaluation of the economic feasibility of the project than Zars. Black’s thesis, ‘A Comprehensive Technical and Economic Feasibility Study of Large-Scale Generation of Electricity by Wind Power at Berlin Pass’, concluded that a wind farm at the pass could repay its capital outlay within a period of approximately 20 years ([2], [11]).

Over the next twenty-one years the project fell by the wayside. Then, in early 2002, a group of four Williams students—Nicholas Hiza, Frederick Hines, Chris Warshaw, and Stefan Kaczmarek, all ’02—became aware of the project in an alternative energies course and decided to learn more about it. Over the next several months, in an effort spearheaded by Hiza, the group completed financial, site, and permitting analyses and found the project to warrant further investigation. A website was created, newspaper articles were written, and suddenly, the Berlin Wind Project came back to life ([11]).

I became involved in the project in June 2003 as the summer ‘wind intern’, during which time I completed the visual impact study (Chapter 9) funded by the Center for Environmental Studies (CES). As the school year began and my focus shifted towards predicting energy yield, my work gradually took the shape of the thesis presented here.

---

Chapter 2

Fluid Flow in Simple Geometries

Ideally, instead of collecting wind data to predict the energy yield at Berlin Pass, we would analytically solve the equations of fluid mechanics to calculate the wind regime there. In practice, however, it is quite difficult to do so—and in order to appreciate this difficulty, we begin with a theoretical examination of fluid flow in planar and cylindrical geometries.

The scenarios presented here represent two of the simplest models of the jet stream, which we assume to generate the wind regime all the way from the ground, where the wind speed is zero, to the altitude of the jet stream itself. I initially intended these models as studies leading up to a fuller two-dimensional characterization of the jet stream flowing over flat ground, with dependence not only on $s$ but on $\theta$ as well (see Figure 2.3). Unfortunately, time constraints did not allow me to complete this more realistic model, and so here I present only the results of the initial studies.

2.1 The Navier-Stokes Equation

The Navier-Stokes equation, which is the general description of the motion of a fluid, is given by

$$\frac{D\vec{v}}{Dt} = -\nabla P + \eta\nabla^2\vec{v} + (\zeta + \frac{1}{3}\eta)\nabla(\nabla \cdot \vec{v}),$$

(2.1)

where $\rho$ is the density of the fluid, $P$ is the pressure, $\vec{v}(\vec{x},t)$ is the velocity of the fluid parcel, and $\eta$ and $\zeta$ are the ‘coefficients of viscosity’ ($\zeta$ is often called the ‘second viscosity’). In general, $\eta$ and $\zeta$ are functions of temperature and pressure, though in most cases they do not vary significantly over the fluid, so we can assume they are constants. In addition, the convective derivative

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}.$$  

(2.2)

If we assume the fluid to be incompressible, then $\nabla \cdot \vec{v} = 0$ and Equation 2.1 reduces to

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{1}{\rho} \nabla P + \frac{\eta}{\rho} \nabla^2 \vec{v}.$$  

(2.3)

1[16], p. 45.
2.2 FLOW BETWEEN PARALLEL PLANES

We will use this simplified version of the Navier-Stokes equation to calculate fluid flow.

2.2 Flow Between Parallel Planes

![Diagram of flow between parallel planes](image)

Figure 2.1: Flow between two parallel planes.

To begin, we consider the simplest motion: steady linear flow between two parallel planes (Figure 2.1). This scenario represents a simple model of the ‘jet stream’ flowing above (and below) the ‘ground’, where the wind speed is defined to be zero. In this case,

\[ \vec{v} = v_x(z) \hat{x}, \]

where the initial condition is \( v_x(0) = v_o \) and the boundary condition is \( v_x(\pm w) = 0 \).

We can now further simplify the Navier-Stokes equation. Because the flow is steady, the derivative of velocity with respect to time is zero. In addition, since each component \( v_i \) does not depend on coordinate \( x_i \), \( (\vec{v} \cdot \nabla)\vec{v} = 0 \) as well.

Note that in Equation 2.3 \( \nabla^2 \vec{v} \) is not the Laplacian because \( \vec{v} \) is a vector. We can, however, expand this term using the the identity

\[ \nabla \times \nabla \times \vec{v} = \vec{v}(\nabla \cdot \vec{v}) - \nabla^2 \vec{v}, \]

which gives

\[ \nabla^2 \vec{v} = -\nabla \times \nabla \times \vec{v} \]

since \( \nabla \cdot \vec{v} = 0 \). Thus,

\[ \nabla^2 \vec{v} = -\nabla \times \left( \frac{\partial v_x}{\partial z} \hat{y} \right) = -\frac{\partial^2 v_x}{\partial z^2} \hat{x}. \]

Finally, if we assume that pressure \( P \) is a linear function of \( x \), then

\[ \nabla P = \frac{\partial P}{\partial x} \hat{x} = k \hat{x}. \]

Combining these results together, the Navier-Stokes equation reduces to

\[ \eta \frac{d^2 v_x}{dx^2} = -k \]
if we drop the $\hat{x}$. This equation can be solved by separation of variables to give

$$v_x(z) = -\frac{k}{2\eta} z^2 - C_1 z - C_2,$$

(2.10)

where $C_1$ and $C_2$ are the constants of integration. Applying our boundary and initial conditions, we obtain

$$v_x(z) = -\frac{k}{2\eta} (z^2 - w^2).$$

(2.11)

This solution tells us that the ‘jet stream’ sets up a parabolic wind speed profile over completely flat ground. If we assume our model jet stream behaves reasonably like the real thing, then its average speed is 41 m/s (92 mph; 300 mph in winter) and its average altitude is 12 km (7.5 mi). Thus, $v_o = 41$ m/s, $w = 1.2 \times 10^4$ m, and we can calculate $k/2\eta$ to be $2.8 \times 10^{-7}$ m$^{-1}$s$^{-1}$. Interestingly, we do not actually have to solve for $k$ or $\eta$—it suffices to calculate their ratio from the boundary conditions. However, since $\eta = 1.8 \times 10^{-5}$ kg/m·sec for air, we can determine that the pressure gradient $k = 1.02 \times 10^{-11}$ N/m$^3$. This particular solution is plotted in Figure 2.2.

![Figure 2.2: The solution to the Navier-Stokes equation for the ‘jet stream’ between two parallel planes ($w = 12,000$ m and $v_o = 41$ m/s).](image)

At the altitude of Berlin Pass (~670 m), the solution gives a wind speed of about 4.4 m/s, which is surprisingly reasonable considering that the average wind speed at Berlin Pass is around 8–10 m/s. Of course, in reality the topography of Berlin Pass is anything but flat, so we cannot rely on this model’s predictions of wind speed to seriously predict a wind farm’s energy yield there.
2.3 Flow Through a Tube

Next, we consider air flow through a tube of radius $R$, where $\vec{v}(\vec{r})$ varies as a function of radius $s$ (Figure 2.3). We can think of this situation as the 'jet stream' flowing above (indeed, through) the ground. Cylindrical coordinates are the most convenient here, so we have

$$\vec{v} = v_z(s) \hat{z},$$

where the initial condition is $v_z(0) = v_o$ and the boundary condition is $v_z(R) = 0$. To simplify the Navier-Stokes equation (Equation 2.3), we must derive $\vec{v} \cdot \nabla$ in cylindrical coordinates; it is not necessarily zero here as it was in the Section 2.2. The algebra is a bit messy, but it turns out that

$$\vec{v} \cdot \nabla = \left( v_s \frac{\partial v_s}{\partial s} + \frac{v_s v_{\phi}}{s} + \frac{v_s v_z}{s} \right) \hat{s} + \left( v_s \frac{\partial v_s}{\partial s} + \frac{v_s v_{\phi}}{s} + \frac{v_s v_z}{s} \right) \hat{\phi} + \left( v_s \frac{\partial v_s}{\partial s} + \frac{v_s v_{\phi}}{s} + v_z \frac{\partial v_z}{\partial z} \right) \hat{z}. \quad (2.13)$$

In our case, $\vec{v} \cdot \nabla = 0$ because the only term that could possibly survive—$\partial v_z / \partial s$—gets multiplied by $v_s$, which is zero. Similarly, $\vec{v}(\vec{r})$ does not depend on time, so $\partial \vec{v} / \partial t = 0$ as well. Thus, Equation 2.3 becomes

$$0 = \nabla P - \eta \nabla \times \nabla \times \vec{v}. \quad (2.14)$$

The second term can be simplified according to Equation 2.5 to become, in the geometry of this scenario,

$$\nabla^2 \vec{v} = \left( \frac{1}{s} \frac{\partial v_z}{\partial s} + \frac{\partial^2 v_z}{\partial s^2} \right) \hat{z} = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial v_z}{\partial s} \right) \hat{z}. \quad (2.15)$$

\[\text{Jet stream data from the National Oceanic and Atmospheric Administration (NOAA) website: http://www.noaa.gov/}.\]

\[\text{[16], p. 46}\]
Finally, if we assume that the pressure $P$ is a linear function of $z$, then $\nabla P = k$ and the Navier-Stokes equation reduces to

$$\frac{d}{ds} \left( s \frac{dv_z}{ds} \right) = \frac{ks}{\eta}$$

(2.16)

if we drop the $\dot{z}$. To solve this equation, we simply integrate twice to obtain

$$v_z(s) = -\frac{k}{4\eta} s^2 + C_1 \ln(s) + C_2$$

(2.17)

where $C_1$ and $C_2$ are constants of integration. We immediately know that $C_1$ must be 0 because if not the function would blow up at the origin. Applying our boundary and initial conditions, we obtain, surprisingly,

$$v_z(s) = \frac{k}{4\eta} (R^2 - s^2).$$

(2.18)

The cylindrical solution differs from the planar solution only by a factor of $1/2$! And, because we fix $R$ and $v_0$ (at 12 km and 41 m/s), the pressure gradient $k$ must increase by a factor of 2 to accommodate—so this solution is exactly the same as that in Section 2.2, except that here $k = 2.03 \times 10^{-11} \text{ N/m}^3$ (see Figure 2.2). Thus, the cylindrical geometry predicts a wind speed of about 4.4 m/s at an altitude of 670 m as well.

### 2.4 The Need for Wind Data

As the calculations above illustrate, to analytically determine air flow over even the simplest geometries can be quite difficult—and as the geometry becomes more complex, the Navier-Stokes equation quickly becomes intractable. Thus, it is not possible to analytically solve it to predict the wind speeds at Berlin Pass; the most we could hope for is to write a computer program to give us a numerical solution.\(^4\) But such a result is at best only an approximation—and hence, the surest way to make long-term predictions of the wind resource at a site is to go out and measure the wind with an anemometer or to use properly adjusted Truewind data (Section 3.2.2).

\(^4\)This is in fact essentially how weather prediction simulations, such as those employed by AWS Truewind, work (see Section 3.2.2).
Chapter 3

Turbines and Prediction Methodology

3.1 Modern Wind Turbines

Since Reed Zars '77 first proposed the idea of installing turbines at Berlin Pass in the late nineteen-seventies, wind technology has advanced tremendously. Whereas his original study recommended the construction of three 200-kW machines, the Berlin Wind Project now proposes a 7–9, 1.5-MW turbine wind farm (10.5–13.5 MW installed capacity). Modern turbines are sleeker, taller, quieter, and more efficient than those available in Zars' day, and consequently they can produce significantly more energy than early models while keeping environmental impact to a minimum. Increased efficiency comes with a commensurate increase in price: Zars' turbines cost a mere $175,000 each (~$530,000 in 2003 dollars;\(^1\) [2]), while General Electric's (GE) 1.5 MW machines come with a $1.05 million sticker price.\(^2\) Though there are other turbines on the market,\(^3\) for the purposes of energy yield calculations this thesis will assume the GE 1.5 MW turbine as default.

3.1.1 Turbine Properties

The proposed General Electric turbines (Figure 3.1) stand 65 meters (213.3 ft) tall ground to hub, with a 3-blade rotor 70.5 m (231.3 ft) in diameter (a 77 m- (252.6 ft-) diameter model is also available). Thus, at the peak of rotation, each blade tip reaches 100.25 m (328.9 ft). The blades rotate at a variable rate between 10.1 and 20.4 rpm, which means that they complete one revolution approximately every 3 seconds.\(^4\) These low rotational speeds are good for two reasons: (1) The only noise they produce is a whooshing sound and (2) in contrast with earlier, faster-turning models, birds have more

\(^2\)Plus an additional $163,000 for installation ([11]).  
\(^3\)Other turbine manufacturers include Vestas and Nordex, both based in Denmark; Suzlon, based in India; and Ecotecnica, based in Spain; for a lengthy list of both small- and large-scale manufacturers, see, e.g., http://www.windustry.com/.  
\(^4\)Helicopter blades rotate much more quickly—on the order of several hundred rpm.
time to see the blades coming and can get out of the way more easily.

The turbines’ cut-in speed—the lowest wind speed at which they will begin turning to produce energy—is 3 m/s (6.7 mph). The cut-out speed, which is the lowest wind speed at which the turbine must shut itself off to avoid structural damage, is 25 m/s (55.9 mph). As the wind accelerates from 3 m/s to 11.8 m/s (26.4 mph), the power output of the turbine rises from 0 MW to its rated capacity of 1.5 MW (Figure 3.4). Above 11.8 m/s, however, the turbine begins to feather its blades in order to ‘spill off’ the extra wind and maintain an output of 1.5 MW; a failure to do so would result in the generator’s producing electricity over its capacity, which could damage it. By 25 m/s, the blades are fully feathered and the turbine ceases to rotate.\(^5\)

As the turbines are over the FAA height limit of 200 ft, they would need to be lighted for aviation safety. However, the lights would be placed on the turbine’s nacelle (the box that sits atop the tower and houses the gears, generator, etc.) as opposed to on the blade tips, so the turbines would reassuringly not look like lighted Ferris-wheels at night. In a phone conversation with Jim Powers of the FAA’s Burlington, VT office,\(^6\) I learned that the FAA recommends—and will perform for free—an aeronautical study of any proposed structure (including a wind farm) in order to evaluate its potential for interfering with flight paths and communications. Additionally, a dialog with local telecommunications authorities would likely be necessary in order to ensure that the proposed farm would not interfere with any signals passing through the airspace of Berlin Pass.

In cold climates such as that of the northwest Berkshires, turbines are subject to

---

\(^5\)Data courtesy of GE Energy.
\(^6\)21 January 2004.
icing. When water droplets in clouds and fog come into contact with turbine blades whose surface is below 0 °C, rime ice can form, changing the aerodynamics of the blades and significantly reducing energy output. When temperatures become warmer and this ice begins to melt, it is shed off the turbine and falls to the ground below, where it can pose a hazard for anyone standing within approximately 250 m (820 ft) upwind of the turbine ([21]; Figure 3.2). Thus, a ~1000-ft ‘safety zone’ would likely need to be established around the wind farm during the wintertime in order to protect users of the Taconic Crest trail system from danger. The Taconic Hiking Club has tentatively approved re-routing the Taconic Crest Trail around the wind farm if the BWP provides the funding to do so ([11]).

Figure 3.2: Rime ice shedding from a turbine. Courtesy of [19].

3.1.2 Power Curves and Mechanical Efficiency

Theoretically, a turbine’s power output is given by\(^7\)

\[
P = \frac{1}{2} C_p \rho A U^3,
\]

where \(\rho\) is the density of the air passing over the rotors, \(A\) is the circular area swept out by the blades as they turn, \(U\) is the wind speed, and \(C_p\) is the dimensionless ‘power coefficient,’ which gives the fraction of the power in the wind that is extracted by the turbine. The theoretical maximum possible value of \(C_p\), called the ‘Betz Limit’ after the German scientist Albert Betz who derived it in 1919\(^8\), is 16/27 = 0.5926 (see Appendix A for derivations of Equation 3.1 and the Betz Limit). This ceiling on power output has

\(^7\)[20]

\(^8\)Published in 1926 in Betz’s Wind-Energie.
3.1. MODERN WIND TURBINES

significant implications for the wind industry: even a perfect turbine can extract only 59.3% of the energy available in the wind.

Real turbines, of course, are not perfect, and their power output is modified by an additional factor of $\eta$, the efficiency, which in general is a function of wind speed $U$ (Figure 3.5):

$$P = \frac{1}{2} C_p \rho A U^3 \cdot \eta(U).$$  \hfill (3.2)

The power output of a real turbine, called the turbine’s ‘power curve’, is plotted along with the theoretical maximum in Figure 3.3. In order to get a better sense of the shape of the real turbine’s power curve, a blow-up version of Figure 3.3 is reproduced in Figure 3.4.

![Power curve comparison](image_url)

Figure 3.3: Theoretical maximum power curve (assuming sea-level air density (1.225 kg/m$^3$), a rotor diameter of 70.5 m, and $C_p = 16/27$) compared to a real power curve for GE’s 1.5 MW turbine. Data courtesy of GE Energy.

In Figure 3.3, we note that the theoretical maximum power output (i.e. the power available in the wind) greatly exceeds the actual output generated by the GE 1.5 MW turbine, especially at wind speeds above about 12 m/s. This behavior is of course due to blade feathering at high speeds. In Figure 3.4, however, we see that between the cut-in (3 m/s) and rated (11.8 m/s) wind speeds the two power curves show the same cubic rise. In the mid-wind-speed regime, then, the GE 1.5 MW turbine produces electricity at almost the theoretically maximum rate; the slight difference in power outputs between the two curves is due entirely to inherent inefficiencies. Thus, if we take the quotient of the two curves, we obtain the turbine’s efficiency, $\eta(U)$ (Figure 3.5).

The sharp rise in efficiency at 3 m/s is due to the turbine suddenly turning on at the cut-in speed, while at high speeds $\eta$ tapers off because of blade feathering. These two effects are the primary contributors to $\eta(U)$’s complicated shape. At moderate wind speeds, when neither the low- nor high-speed effects are important, $\eta$ is not 1 only because of mechanical inefficiencies (such as friction) in the gears, generator, etc. Thus, the GE 1.5 MW turbine’s inherent mechanical efficiency $\eta_{\text{mech}}$ is probably around the
3.1. MODERN WIND TURBINES

Figure 3.4: A close-up view of the GE 1.5 MW power curve from Figure 3.3.

Figure 3.5: The efficiency of GE’s 1.5 MW turbine. The peak of .76 at about 10 m/s probably represents the mechanical efficiency $\eta_{mech}$. 
maximum value of $\eta(U)$: 0.76. While 76% efficiency is quite good compared to the $\sim 30\%$ of a gasoline engine, perhaps further engineering advances can push mechanical efficiency closer to 1.

### 3.2 Energy Production

To predict energy production over a period of, say, of one month, we first need a distribution (a.k.a. histogram) of wind speeds for the site. If this distribution has $n$ bins with $c_i$ counts in each bin, we can calculate the energy yield as follows:

$$E = \sum_{i=0}^{n} c_i \cdot P_{i,j} \cdot t, \quad (3.3)$$

where $i$ is the bin number and $P_{i,j}$ is the power output for bin $i$, which is determined by the turbine’s $j^{th}$ power curve.$^9$ The time interval $t$ is the amount of time over which wind speeds were averaged to produce each count.$^{10}$

In practice, each of the quantities that go into Equation 3.3 can be obtained in several different ways. Speed distributions, for example, can be generated either from wind data or from wind models. Additionally, for those generated from data, wind speeds measured relatively close to the ground (e.g. 40 m) can be extrapolated upwards to the hub height (e.g. 65 m) using several different methods. Finally, the particular power curve $P_{i,j}$ depends on air density. Though energy yield is calculated using Equation 3.3 in each case, the details differ; in the remainder of this chapter we examine these various permutations and flesh out a more complete methodology.

### 3.2.1 Speed Distributions

**Empirical Distributions**

The most obvious way to obtain a wind speed distribution is to measure the winds at a site over some period of time and then generate a histogram from them. An example of such a histogram, which shows the distribution of wind speeds at Brodie Mountain (Lanesboro, MA) in January, 1998 is presented in Figure 3.6.

Though not shown, the number of counts in the bin at $U = 0$ is enormous: $\sim 1100$. While it is possible that the wind averaged 0 m/s this frequently over the course of the month, a more plausible explanation is that the anemometers were often obstructed from turning. Such anemometer obstruction can be caused by a number of factors, including (1) the loss of contact between the instrument's electrodes and the wire running to the data logger, (2) debris like leaves, grasses, dust, etc. getting caught or tangled in the cups, and (3) rime ice building up on the cups and literally freezing them in place. Of all these possible hindrances, it is icing that probably occurs most frequently in cold conditions.

---

$^9$Each turbine has a set of power curves whose shape depends on ambient air density; see Section 3.2.4.

$^{10}$Many anemometers measure wind speeds for 10 minutes and then record only the average over this interval. For these instruments, $t = 10 \text{ min} = 600 \text{ sec}$. 

3.2. ENERGY PRODUCTION

Figure 3.6: The distribution of wind speeds at Brodie Mountain (Lanesboro, MA) in January 1998. Data courtesy of UMass’ Renewable Energy Research Laboratory (RERL).

climates such as that of western Massachusetts. Hence, the outlier at 0 m/s is likely due predominantly to icing events.

In order to account for these ‘lost’ data intervals in calculating the total energy yield, we must take the result of Equation 3.3 and modify it slightly. To do so, we will assume that the ‘missing’ data would have been distributed proportionately throughout the histogram if we had been able to record it. Thus,

$$E_{\text{tot}} = E + \frac{E}{t} \cdot c_0 = E \cdot \left(1 + \frac{c_0}{t}\right),$$

(3.4)

where $E$ is the energy without the missing intervals, $t$ is the time interval, and $c_0$ is the number of ‘missing counts’ from bin 0. Although we do not actually know how the wind speeds were distributed during the missing intervals, this approximation is reasonable. The larger percentage of data that are missing, however, the shakier this assumption becomes.

**Weibull Distribution**

The Weibull distribution is a probability distribution that has been found to approximate annual wind speed distributions well at sites with various wind conditions. The distribution is given by

$$p(U) = \left(\frac{k}{c}\right) \left(\frac{U}{c}\right)^{k-1} e^{-\left(\frac{U}{c}\right)^k},$$

(3.5)

where $c$, the ‘scale parameter’, is related to the mean wind speed and $k$, the ‘shape parameter’, describes the variability about the mean. Larger values of $k$, such as 2.5 or 3, signify a small amount of variation about the mean, while smaller values of $k$ around 1.2 or 1.5 mean that wind speeds are more variable\(^{11}\) (see Figure 3.7). The cumulative

\(^{11}\)After [6], p. 14-6 and [20], p. 57-60.
distribution function, which gives the area under the Weibull curve \( p(U') \) from \( U' = 0 \) to \( U' = U \), is:
\[
F(U) = 1 - e^{-(U/c)^k}.
\]  
(3.6)

If we compare Figure 3.7 to Figure 3.6, we see that a Weibull distribution with \( k \approx 2.3 \) approximates the measured speed distribution at Brodie Mountain fairly well.

![Weibull distributions](image)

Figure 3.7: Weibull distributions of wind speed generated from Equation 3.6 for \( k = 2.3 \) and \( k = 1.2 \) (\( c = 10 \) in both cases). The distributions have been multiplied by an arbitrary factor of 4,464 (the number of 10-minute intervals in a month with 31 days) so that the scale is similar to that in Figure 3.6.

**Rayleigh Distribution**

The Rayleigh distribution is a special case of the Weibull distribution where \( k = 2 \). Though the Rayleigh distribution can be expressed using \( c \) and \( k \) (or rather, \( c \) and \( 2 \)), it is also easily expressible in terms of the mean wind speed \( \bar{U} \). Since it is much easier to measure or predict \( \bar{U} \) than \( c \) and \( k \), and since 2 has been found to be a typical \( k \) value at many locations, the Rayleigh distribution is preferred to the Weibull distribution in cases where \( \bar{U} \), as opposed to \( c \) and \( k \), are available. In addition, wind turbine manufacturers often quote standard turbine performances using the Rayleigh distribution. Nevertheless, the full Weibull distribution’s extra parameter makes it the preferred choice if \( c \) and \( k \) values can be obtained. The Rayleigh distribution, expressed in terms of mean wind speed, is given by
\[
p(U) = \frac{\pi}{2} \left( \frac{U}{\bar{U}^2} \right) e^{-\frac{U}{\bar{U}}}.
\]  
(3.7)

The cumulative distribution function, which gives the area under the Rayleigh curve \( p(U') \) from \( U' = 0 \) to \( U' = U \), is
\[
F(U) = 1 - e^{-\frac{U}{\bar{U}^2}}.
\]  
(3.8)
3.2. ENERGY PRODUCTION

In Figure 3.8, we see that a Rayleigh distribution with $\bar{U} \approx 10$ approximates the measured speed distribution at Brodie Mountain (Figure 3.6) fairly well, just as the Weibull distribution does in Figure 3.7. At the Brodie site, then, the two theoretical distributions seem to be more or less equivalent—though the Weibull distribution is probably a little better since $c$ and $k$ values are available.

![Rayleigh distributions of wind speed generated from Equation 3.8 for $\bar{U} = 10$ and $\bar{U} = 20$. The distributions have been multiplied by an arbitrary factor of 4,464 so that the scale is similar to that in Figure 3.6.]

3.2.2 Truewind

In order to use the Weibull and Rayleigh distributions to predict the wind speeds, and hence the energy that could be produced, at a given location, we need $c$, $k$, and $\bar{U}$ values specific to the site in question. AWS Truewind, LLC, an energy technology and atmospheric modeling firm based in Albany, NY, offers free web access to wind resource maps from which these parameters may be obtained.\(^{12}\) Additionally, Truewind has produced a geographical-information-systems (GIS) data layer version of the New England wind map that may be requested from the Massachusetts Technology Collaborative (MTC).\(^{13}\)

The maps, for the most part funded by government, industry, and academic agencies, are created by AWS Truewind using a proprietary program called MesoMap\(^ {TM}\) that uses high-resolution gridded atmospheric weather data—as opposed to surface wind speed measurements—and a weather-modeling program called MASS (Mesoscale Atmospheric Simulation System) to accurately model the wind over large geographical areas.\(^ {14}\) Over the course of the next several chapters, we will compare Truewind predictions of energy production to those made from anemometer data so that we can determine just how good Truewind estimates are in the NW Berkshire region. In Chapter 7 we will use this knowledge to carefully derive an estimate of energy yield at Berlin Pass.


\(^{13}\)Massachusetts Technology Collaborative (MTC): http://www.mtpc.org/.

\(^{14}\)For a detailed description of MesoMap, see [4].
3.2. ENERGY PRODUCTION

Truewind’s online maps provide quite a wealth of information: the New York state
map, for example, has a spatial grid resolution of 400 m × 400 m and gives c and k values,
average speed, and average power density for each of the four seasons at elevations of 30
m, 65 m, and 100 m off the ground, as well as c and k values, wind speed frequency, and
power distributions for sixteen separate compass directions (22.5° sectors)—in addition
to providing a wind rose diagram. The New England wind map has a grid resolution of
200 m × 200 m but only provides seasonal c and k values for 50 m elevation; annual c
and k values are given for 30 m, 50 m, 70 m, and 100 m. The GIS map provides similar
information.

3.2.3 Speed Extrapolation

Winds lower to the ground flow more slowly than winds higher up in the atmosphere
because vegetation, topography, etc. cause drag that slows low winds down. In general,
then, wind speed increases with height in some complicated and turbulent way depending
on local conditions and topography. Nevertheless, two ‘velocity extrapolation laws’—the
‘log law’ and the ‘power law’—have been found to approximate the wind speed profile
well in many situations.\textsuperscript{15} These laws can be used to predict the wind speed at the height
of power generation (e.g. 65 m) from wind speeds measured at a lower height (e.g. 40
m).

Log Law

The log law, which can be derived theoretically using several different methods,\textsuperscript{16} is
given by

\[ U(z) = U(z_r) \frac{\ln(z/z_o)}{\ln(z_r/z_o)}, \]

(3.9)

where \( U(z) \) is the speed at the height of power generation, \( U(z_r) \) is the speed at the
height of measurement \( z_r \), and \( z_o \) is ‘roughness length’. Roughness lengths range from
.01 mm for wind flowing over smooth ice or mud, to 10 mm over rough pasture, to
0.5 m over forests and woodlands.\textsuperscript{17} In order for Equation 3.9 to produce reasonable
results, the reference height \( z_r \) can neither be 0 nor equal to the surface roughness \( z_o \),
since the expression is undefined in these cases. Normally, this restriction does not cause
any problems, since wind speeds are never measured at the ground and values of \( z_o \) are
generally two or three orders of magnitude smaller than values of \( z_r \). The log law speed
profile is plotted along with the power law profile in Figure 3.9.

\textsuperscript{15}The log and power laws predict wind speeds best over flat terrain but do a reasonable job over
‘complex’ terrain—such as that at Berlin Pass—as well. [20]

\textsuperscript{16}E.g. boundary layer flow, mixing length theory, eddy viscosity theory, similarity theory; see [20], p.
42.

\textsuperscript{17}For a complete table of values, see [20], p. 44.
3.2. ENERGY PRODUCTION

Figure 3.9: Comparison of log and power law extrapolations of wind speed above a reference height. Here, $U(z_r) = 10$ m/s, $z_r = 40$ m, and $z_o = 0.5$ m. Note that at 25 m above $z_r$, the log and power law predictions differ by 0.4 m/s or 11.3%

**Power Law**

The purely empirical power law is given by

$$U(z) = U(z_r) \cdot (z/z_r)^\alpha, \quad (3.10)$$

where $\alpha$ is known as the ‘power law exponent’. Particularly in flow over flat planes, $\alpha$ takes the value of $1/7$—but in practice it varies with elevation, time of day, season, topography, wind speed, temperature, and other factors.\(^{18}\) Because of this variability, the log law, instead of the power law, is generally used to extrapolate wind speeds.

The log and power law profiles are compared in Figure 3.9, where the reference height $z_r = 40$ m, the roughness coefficient $z_o = 0.5$ m, $U(z_r) = 10$ m/s, and $\alpha = 1/7$. The log law consistently predicts higher wind speeds than the power law, with a difference of 0.4 m/s at 25 m above $z_r$. This difference is equivalent to a quotient of 1.04, which means that the log law predicts that $(1.04)^3 - 1 = 11.3\%$ more energy could be generated than does the power law. Thus, the choice of velocity extrapolation law is not insignificant with respect to predicted energy yield.

### 3.2.4 Air Density

As we know from Equation 3.1, the power output, and hence energy yield, depends on air density $\rho$. In practice, however, we calculate energy using Equation 3.3, which does not contain an overt $\rho$ term. The reason is that $\rho$ hides inside $P_{t,j}$—the turbine’s power curve—where $j$ denotes the power curve for a particular value of ambient air density. Thus, GE provides *eleven* separate power curves for densities ranging from 1.02 kg/m\(^3\).\(^{18}\)[20], p. 45.
to 1.225 kg/m³ (sea level), with a resolution of 0.02 kg/m³. In order to determine which
power curve to use, we must calculate ρ at the height of the turbine’s hub.

In order to do so, we require two quantities: a temperature and a pressure. These
quantities can be combined according to Equation 3.11 to give air density:\(^19\)

\[ \rho(z) = \left( \frac{P_o}{RT} \right) e^{-gz/RT}, \quad (3.11) \]

where \( T \) is the air temperature, \( P_o \) is the air pressure at some known reference altitude
(such as 101,325 Pa at sea level), \( z \) is the distance above that reference altitude, \( R \) is
the gas constant (286.4 J/kg-K)\(^20\), and \( g \) is the acceleration due to gravity (9.8 m/s\(^2\)).
However, since \( \rho \) varies in a non-linear way with height \( z \), it is better to calculate the
average air density over the height of the rotor\(^21\):

\[ \bar{\rho} = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} \rho(z)\,dz \]

\[ = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} \left( \frac{P_o}{RT} \right) e^{-gz/RT}\,dz \]

\[ = \frac{1}{h_2 - h_1} \frac{P_o}{g} \left( e^{-gh_2/RT} - e^{-gh_1/RT} \right), \quad (3.14) \]

where \( h_1 \) and \( h_2 \) are the heights of the bottom and top of the rotors, respectively.

Unfortunately, it is difficult to obtain temperatures and pressures at a given site
because not all anemometer towers are equipped with thermometers and barometers. In
practice, then, to calculate \( \rho \) we must collect these quantities from a proxy site and then
extrapolate their values to the site in question.

Proxy temperatures may be obtained from, among other sources, archived weather
data from a set of four weather stations in Williams College’s 2,500-acre Hopkins Memori-
al Forest (HMF).\(^22\) These data provide a good proxy for temperatures at Berlin Pass,
since the HMF is only 2–3 miles north of the Pass and is quite similar to it in topography,
vegetation cover, and weather patterns. However, since the HMF weather stations and
the proposed turbines lie at different elevations and air temperature generally decreases
with increasing altitude, to obtain a reasonable estimate of the temperature at the Pass
we must extrapolate temperatures upwards. We can do so in the following manner:

\[ T(z) = T(z_o) - l(z - z_o), \quad (3.15) \]

\(^19\)Oklahoma Wind Power Tutorial Series, Lesson 2', accessed 2.18.04:
http://www.seic.okstate.edu/cwpi/about/Library/Lesson2_2airdensity.pdf.
\(^20\)The CRC Handbook of Chemistry and Physics ([17]) gives \( R = 8.314 \) J/mol·K. It also states that
dry air at STP (0 °C, 1 atm) has a density of 1.296 g/L. Given that, at STP, 1 mole of gas occupies
22.4 L, the mass per mole of air turns out to be 0.02903 kg/mol. Thus, dividing \( R \) by this value gives
\( R = 286.39 \) J/kg·K for dry air at STP.
\(^21\)A better approximation would be to calculate the average density across the circular face of the
rotor.
\(^22\)The HMF is operated by Williams’ Center for Environmental Studies (CES). Historical
weather data is available for the period January 1983–December 2000 on the HMF website at
http://www.williams.edu/CES/hmf/ (accessed 10.9.03).
where \( T(z) \) is the temperature at the height of the turbines, \( T(z_0) \) is the temperature at a reference height \( z_0 \), and the standard atmospheric temperature lapse rate \( l = 0.0065 \degree C/m \). Note that since this approximation is linear there is no need to calculate the average temperature across the face of the rotor.

Though the HMF weather stations are equipped with barometers, a cursory look at their pressure data suggests that they may be functioning improperly. The National Climatic Data Center (NCDC), a department of the National Oceanographic Atmospheric Administration (NOAA), maintains an online archive of climatological data from a network of weather stations around the country, including one at Harriman and West Airport in North Adams, MA (station KAQW; 42°40’ N, −73°10’ W).\(^{24}\) Archived data include daily and monthly averages for temperature, pressure, wind speed, etc., and are available for the period November 1996–Present.\(^{25}\) The pressure values from this station, along with temperatures extrapolated from HMF data (or from KAQW data, if no HMF temperatures are available), can be plugged into Equation 3.14 to estimate the air density, thereby determining which power curve \( P_{ij} \) to use. Alternatively, we may plug this calculated density into Equation 3.1 to calculate the theoretical maximum power curve, which gives the total power available in the air.

### 3.2.5 Error Analysis

We have now described a number of different ways to collect the pieces that go into Equation 3.3. No estimate of energy yield is complete, however, without a concurrent estimate of the error in that figure. While the particulars of error analysis are left to subsequent chapters, a general methodology for calculating uncertainty is outlined here.

Consider a function \( f \) that depends on the measured quantities \( x, \ldots, z \). The error in \( f \), \( \delta f \), is simply the addition in quadrature of the errors in \( x, \ldots, z \) and the partial derivatives of \( f \) with respect to these variables ([24], p. 73). That is,

\[
\delta f(x, \ldots, z) = \sqrt{ \left( \frac{\partial f}{\partial x} \delta x \right)^2 + \cdots + \left( \frac{\partial f}{\partial z} \delta z \right)^2 }.
\]

Thus, for example, the error in the log-law-extrapolated wind speed \( U \) that is due to uncertainty in the measured wind speed \( U_r \) and roughness coefficient \( z_0 \) (see Equation 3.9), is

\[
\delta U = \sqrt{ \left( \frac{\partial U}{\partial U_r} \delta U_r \right)^2 + \left( \frac{\partial U}{\partial z_0} \delta z_0 \right)^2 }.
\]

Over the course of Chapters 4–6, we will use this method to determine the error in energy yields predicted in the various manners described above.

---

\(^{23}\) In reality, \( l \) is a variable quantity that depends on humidity, altitude, etc. Future research could include the calculation of a better value of \( l \) for the northwest Berkshire region.


\(^{25}\) At the time of this writing, the range is November 1996–April 2004.
Chapter 4

Brodie Data Analysis

The first of the proxy data sets that we will use to explore the relationship between theoretical and empirical estimates of energy production—and then to predict energy yield at Berlin Pass—is a collection of wind data gathered at Brodie Mountain (Lanesboro, MA; 42°36′ N, 73°16′ W). During the period November 1996–August 1999, the Renewable Energy Research Laboratory (RERL) at UMass Amherst maintained an anemometer tower on top of the former ski area, some 14.4 km (8.9 mi) southeast of Berlin Pass (Figure 4.1). Fixed to the tower, whose base stood at approximately 790 m above sea level, were five (calibrated) instruments: three NRG Systems ‘#40 Maximum’ anemometers at heights of 10 m, 25 m, and 40 m above the ground and two NRG 200-Series wind vanes at heights of 25 m and 40 m. At the base of the tower was an NRG 9300 data logger that sampled speed and directional data once per second and recorded them as 10-minute averages. U. Abdulwahid of RERL reformatted the raw data in 2001 and posted them to the lab’s website.

4.1 Energy Production Estimates

Though the Brodie data cover a period of almost three years, complete data sets exist for only 1997 and 1998. In this chapter, then, we examine only these sets, focusing primarily on the data from 1 January–31 December 1998. Using various combinations of the methods outlined in Chapter 3, we estimate annual energy production (and margin of error) for a 7-turbine wind farm at Brodie Mountain in six separate ways, including:

1. The log law
2. The power law, where the ‘power exponent’ \(\alpha\) is equal to 1/7
3. The power law, where \(\alpha\) is calculated separately for each time interval

\(^1\)The wind speed data from three separate heights could be used to test whether the log and power laws accurately model the wind speed profile over ‘complex’ terrain. Because of time constraints, however, I did not have time to carry out this analysis—I leave the calculations to a future student.

\(^2\)Renewable Energy Research Laboratory (RERL), UMass Amherst: http://www.ecs.umass.edu/mie/labs/rerl/.
Figure 4.1: The location of Brodie Mountain with respect to Berlin Pass. The approximate site of UMass’ anemometer is marked towards the bottom of the map. The proposed site of the BWP is located near the top.
4. The Weibull distribution

5. The Rayleigh distribution

6. We also calculate the total energy available in the wind to provide a comparison for the five production estimates.

In Section 4.2, we will compare the results to determine which methods give the most reasonable estimates of energy yield.

### 4.1.1 Log Law

#### Energy Estimate

In order to use the log law to estimate energy production at Brodie Mountain, we must first determine the average monthly air densities at the site so that we may choose the power curve appropriate to each month. The HMF temperature and KAQW pressure data\(^3\) for 1998, and the monthly air densities calculated from them, are presented in Table 4.1.

Table 4.1: Average monthly air temperatures (HMF), pressures (KAQW), and densities (calculated) for Brodie Mountain in 1998, which together determine which power curve \(P_{i,j}\) to use for each month’s energy estimate.

<table>
<thead>
<tr>
<th>Month</th>
<th>HMF Temp, °C</th>
<th>KAQW Press, in Hg</th>
<th>(\bar{\rho}), kg/m(^3)</th>
<th>Power Curve (P_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>−1.94</td>
<td>29.32</td>
<td>1.193</td>
<td>1.20→ (j = 10)</td>
</tr>
<tr>
<td>Feb</td>
<td>−0.91</td>
<td>29.22</td>
<td>1.184</td>
<td>1.18→ (j = 9)</td>
</tr>
<tr>
<td>Mar</td>
<td>2.49</td>
<td>29.23</td>
<td>1.171</td>
<td>1.18→ (j = 9)</td>
</tr>
<tr>
<td>Apr</td>
<td>8.01</td>
<td>29.17</td>
<td>1.147</td>
<td>1.14→ (j = 7)</td>
</tr>
<tr>
<td>May</td>
<td>15.46</td>
<td>29.15</td>
<td>1.119</td>
<td>1.12→ (j = 6)</td>
</tr>
<tr>
<td>Jun</td>
<td>17.12</td>
<td>29.09</td>
<td>1.111</td>
<td>1.12→ (j = 6)</td>
</tr>
<tr>
<td>Jul</td>
<td>19.38</td>
<td>29.20</td>
<td>1.107</td>
<td>1.10→ (j = 5)</td>
</tr>
<tr>
<td>Aug</td>
<td>17.24</td>
<td>29.28</td>
<td>1.117</td>
<td>1.12→ (j = 6)</td>
</tr>
<tr>
<td>Sep</td>
<td>12.74</td>
<td>29.17</td>
<td>1.130</td>
<td>1.12→ (j = 6)</td>
</tr>
<tr>
<td>Oct</td>
<td>9.60</td>
<td>29.33</td>
<td>1.148</td>
<td>1.14→ (j = 7)</td>
</tr>
<tr>
<td>Nov</td>
<td>3.39</td>
<td>29.26</td>
<td>1.169</td>
<td>1.16→ (j = 8)</td>
</tr>
<tr>
<td>Dec</td>
<td>0.74</td>
<td>29.31</td>
<td>1.181</td>
<td>1.18→ (j = 9)</td>
</tr>
</tbody>
</table>

We can now use the log law to extrapolate the measured wind speeds from 40 m to 65 m and apply Equation 3.3 to obtain monthly and annual energy yields for a hypothetical 7-turbine wind farm at Brodie Mountain. The results of these calculations are presented in Table 4.2.

\(^3\)See Chapter 3.
Table 4.2: Monthly and annual estimates for energy production at Brodie Mountain made using the log law. The predicted annual total is 149.4% of Williams College’s 2002-2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>3.361</td>
</tr>
<tr>
<td>Feb</td>
<td>2.454</td>
</tr>
<tr>
<td>Mar</td>
<td>3.265</td>
</tr>
<tr>
<td>Apr</td>
<td>1.966</td>
</tr>
<tr>
<td>May</td>
<td>2.630</td>
</tr>
<tr>
<td>Jun</td>
<td>2.179</td>
</tr>
<tr>
<td>Jul</td>
<td>1.846</td>
</tr>
<tr>
<td>Aug</td>
<td>1.241</td>
</tr>
<tr>
<td>Sep</td>
<td>2.028</td>
</tr>
<tr>
<td>Oct</td>
<td>3.369</td>
</tr>
<tr>
<td>Nov</td>
<td>3.977</td>
</tr>
<tr>
<td>Dec</td>
<td>3.769</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td><strong>32.09</strong></td>
</tr>
<tr>
<td><strong>% Wms. Col.</strong></td>
<td><strong>149.3%</strong></td>
</tr>
</tbody>
</table>

Figure 4.2: Monthly log law energy production estimates for a 7-turbine wind farm at Brodie Mountain, 1998.
These results illustrate an important fact about the wind resource in the Berkshire region and much of the rest of the northeastern United States: more wind is available during the winter months than during the summer. In fact, at Brodie Mountain in 1998 almost twice as much energy could have been produced during the windiest month (November) than during the least windy month (August). Unfortunately, monthly wind power production does not perfectly match electricity demand in New England, as data taken from the NEPOOL (New England Power Pool) website\(^4\) illustrate in Figure 4.3. There seem to be two seasonal peaks in energy demand—during the coldest part of the winter and the hottest, most humid part of the summer—with periods of lower demand in between. This wind-derived electricity could be useful in helping to meet demand during the more blustery wintertime, but it would would more difficult for it to do so during the calmer summer months.

![Figure 4.3: Monthly energy demand in New England, March 2003–February 2004. Data courtesy of ISO New England.](image)

Electricity demand at Williams College for the period July 2002–June 2003 is presented in Figure 4.4. During this academic year, the college used approximately 21.5 million kW-hr, which cost $1.82 million.\(^5\) The predicted 32.1 million kW-hr from a wind farm at Berlin Pass represents a full 149.3% of Williams College’s annual electricity use.

**Uncertainty**

To estimate the uncertainty in the 32 million kW-hr figure, we must first identify the uncertainties in all the variables that go into the log law calculation. Since air density \(\rho\) depends on measured values of temperature \(T_0\) (from HMF), pressure \(P_0\) (from KAQW), altitude \(z\) (from ArcGIS data), and the atmospheric temperature lapse rate \(l\), we can use

---


\(^5\)Reported by Williams College’s Department of Buildings and Grounds.
Equation 3.16 to determine the uncertainty in \( \rho \). And since density determines which power curve \( P_j \) we use, the uncertainty in these quantities conspire to give a ‘power-curve uncertainty’—that is, a range of reasonable \( j \) values. Additionally, uncertainty in the wind-speed measurements made by UMass’ anemometers, as well as that in the value of \( z_o \), the roughness coefficient, affect the final uncertainty in the annual energy production figure.

The error in \( \rho(T_o, P_o, z, l) \) is given by

\[
\delta \rho = \sqrt{\left( \frac{\partial \rho}{\partial T_o} \delta T_o \right)^2 + \left( \frac{\partial \rho}{\partial P_o} \delta P_o \right)^2 + \left( \frac{\partial \rho}{\partial l} \delta l \right)^2 + \left( \frac{\partial \rho}{\partial z} \delta z \right)^2}, \tag{4.1}
\]

where

\[
\rho(T_o, P_o, z, l) = \left( \frac{P_o}{R(T_o - l(z - h_o))} \right) e^{-g(z-h_1)/R(T_o-l(z-h_o))} \tag{4.2}
\]

and the gas constant \( R = 286.39 \text{ kg/mol-K} \), the reference altitude of the HMF anemometer at which \( T_o \) is measured is \( h_o = 266 \text{ m} \), \( g = 9.8 \text{ m/s}^2 \), and the quantity \( R(T_o-l(z-h_o)) \) is the temperate at the hub altitude extrapolated from HMF temperature. For April 1998, a ‘representative’ month with only 0.3% of data intervals missing, \( T_o = 4.2^\circ \text{C} = 277.2 \text{ K} \) while \( P_o = 98,781 \text{ Pa} \) at the altitude of the airport \( h_1 = 196.1 \text{ m} \). In addition, the altitude of the turbine hubs is \( z = 855 \text{ m} \) and the lapse rate is assumed to be the standard \( l = 0.0065^\circ \text{C}/\text{m} \).

Taking the four partial derivatives, we obtain:

\[
\frac{\partial \rho}{\partial z} = \frac{P_o \cdot e^{-g(z-h_1)/R(T_o-l(z-h_o))}}{R(T_o - l(z - h_o))^2} \left( l - \frac{g(T_o + l h_o)}{R(T_o - l(z - h_o))} \right) \tag{4.3}
\]

\[
\frac{\partial \rho}{\partial T_o} = \frac{P_o \cdot e^{-g(z-h_1)/R(T_o-l(z-h_o))}}{R(T_o - l(z - h_o))^2} \left( 1 + \frac{g(z-h_1)}{R(T_o - l(z - h_o))} \right) \tag{4.4}
\]
\[ \frac{\partial \rho}{\partial l} = \frac{P_o(z-h_o)e^{-g(z-h_1)/R(T_o-l(z-h_0))}}{R(T_o-l(z-h_0))^2} \left( 1 - \frac{g(z-h_1)}{R(T_o-l(z-h_0))} \right) \] (4.5)

\[ \frac{\partial \rho}{\partial P_o} = \frac{e^{-g(z-h_1)/R(T_o-l(z-h_0))}}{R(T_o-l(z-h_0))} \] (4.6)

The NCDC reports average monthly temperatures to a precision of \( \delta T_o = \pm 1^\circ F = \pm 5/9^\circ C \) and average monthly pressures to \( \delta P_o = \pm 1 \) in Hg = \( \pm 33.86 \) Pa. There is a range of \( l \) values: while the standard lapse rate is .0065\(^\circ\)C/m, the dry adiabatic lapse rate is .0098\(^\circ\)C/m.\(^6\) If we split the difference between these two, then \( \delta l = \pm .00165^\circ\)C/m.

There is also a range of values for \( z \): the upper limit, measured from an ArcGIS data layer, puts the base of the tower at 790 m (and hence the hub, 65 m off the ground, at 855 m), while the lower limit, reported by [23], has the tower at an elevation of 761 m. To estimate the uncertainty in altitude, I simply split the difference between the two values and rounded up to obtain \( \delta z = \pm 15 \) m.

Finally, combining all this information according to Equation 4.1 tells us that, for April 1998, the uncertainty in air density \( \delta \rho = \pm .0048 \) kg/m\(^3\). To make sure April is adequately representative, I ran the calculations for several other months\(^7\) and obtained uncertainties in the range .0044 to .0051 kg/m\(^3\). The small magnitude of \( \delta \rho \) is significant: since the difference in air densities for two adjacent power curves (e.g., the curve for 1.20 kg/m\(^3\) air and that for 1.18 kg/m\(^3\) air) is .02 kg/m\(^3\), the uncertainty in \( \rho \) is not large enough to bump us—except in rare cases when a \( \rho \) value sits right on the cusp between two curves—onto a different power curve. Thus, the uncertainties in temperature, pressure, temperature lapse rate, and altitude are not large enough to affect the final energy estimate. We can conclude, then, that the dominant uncertainty here is in \( P_{i,j} \) and is due to the finite resolution (0.2 kg/m\(^3\)) between the power curves.

Uncertainties in the measured wind speed \( U_r \) and the value of the roughness coefficient \( z_o \) come into play when extrapolating the wind speeds upwards with the log law.\(^8\) The partial derivatives of the log law with respect to these variables are:

\[ \frac{\partial U}{\partial U_r} = \frac{\ln(z/z_o)}{\ln(z_r/z_o)} \] (4.7)

\[ \frac{\partial U}{\partial U_r} = \frac{U_rz_o}{(\ln(z_r/z_o))^2}(\ln(z/z_o) - \ln(z_r/z_o)) \] (4.8)

NRG Systems reports that its #40 Maximum anemometers record wind speeds with a precision of 1%, so the uncertainty in \( U_r \) depends on wind speed (\( \delta U_r = \pm .1 \) m/s for 10 m/s wind, \( \pm .2 \) m/s for 20 m/s, etc.). Manwell et al. give a roughness length of 0.5 m

---

\(^6\)To make sure \( l = .0065^\circ\)C/m is a reasonable daily value to use, I obtained average daily temperatures from Harriman and West Airport (KAQW; elev. 196.9 m) and Hopkins Forest (HMF; elev. 266 m) for 5 representative days (11/13/1996, 6/25/1997, 3/7/1998, 9/18/1999, and 1/16/2000). Extrapolating the AQW temperatures upwards to HMF produced an average temperature discrepancy of only 0.29\(^\circ\)C (\( \sigma = 1.1^\circ\)C). Such a small temperature discrepancy justifies the use of \( l = .0065^\circ\)C/m in our calculations.

\(^7\)January, July, and September 1998.

\(^8\)We ignore the uncertainty introduced by the ‘accuracy’ of the log law in modeling the wind speed profile over complex terrain; a more complete error analysis would take this factor into account as well.
for “forest and woodlands”, while Burton et al. give 0.7 m for “cities, forests” and 0.3 m for “suburbs, wooded countryside”, so using \( z_0 = 0.5 \) m and \( \delta z_0 = \pm 0.2 \) m seems reasonable. Also, \( z = 65 \) m and \( z_r = 40 \) m. Combining this information together using

\[
\delta U = \sqrt{\left( \frac{\partial U}{\partial U_r} \delta U_r \right)^2 + \left( \frac{\partial U}{\partial z_0} \delta z_0 \right)^2},
\]

we find that \(| \delta U | > 0.25 \) m/s for \( U_r > 21.5 \) m/s. Since the resolution of GE’s power curves is 0.5 m/s, when we calculate the empirical speed distribution using the log law, data points can only be placed in the wrong bins if they are derived from measured speeds greater than 21.5 m/s (since an error of \( 0.5 / 2 = 0.25 \) m/s necessarily overlaps bins). Any data point derived from a measured speed lower than 21.5 m/s, therefore, will be placed in the correct bin and so cannot produce uncertainty in the final energy estimate. At speeds greater than about 13.5 m/s, however, the power curves are completely flat, since the turbines reach their rated capacity of 1.5 MW at such high speeds. Thus, even though some data points could potentially be misplaced because of the error in \( U \), the uncertainty in energy produced is zero because the turbines produce the same power regardless of whether the data points fall into one high-speed bin or its neighbor. Therefore, the dominant uncertainty in \( P_{i,j} \) is also due to its 0.5 m/s bin size.

To summarize, the errors in temperature, pressure, atmospheric temperature lapse rate, and altitude are not great enough to produce uncertainty in which power curve we use. In addition, the errors in measured wind speed and roughness length do not produce great enough uncertainty in extrapolated wind speeds to affect on the final energy estimate. Therefore, given the stated uncertainties in the relevant quantities, the density-resolution of 0.02 kg/m³ between power curves \( P_j \), the bin size of 0.5 m/s for those curves—and if we assume the log law correctly predicts wind speed—the energy production estimate has error bars of zero. Thus, a 7-turbine wind farm on Brodie Mountain in 1998 could have produced 32.1 ± 0 million kW-hr of energy.

This is not to say that there is no uncertainty in the final energy estimate. Rather, what the zero error bars signify is that, given the power curves \( P_{i,j} \) that GE provides, the dominant uncertainty is in the power curves themselves. Unfortunately, since GE does not provide uncertainty estimates \( \delta P_{i,j} \) with its power curves, there is no way to know the magnitude of this error—and so we ignore it here. In addition, there is an uncertainty in the counts \( c_i \) themselves that is due to the large number of counts in the bin for wind speeds of zero (bin \( c_0 \)). We assumed that these counts should have been distributed proportionately across the histogram, but in reality this might not have been the case. Unfortunately, there is no way to know how those counts should have been distributed—and so the magnitude of this error is unknown as well. A more complete error analysis, then, would also take these two uncertainties into account. Finally, the above result does not mean that the wind farm would produce 32.1 million kW-hr every year. Instead, it signals that the year-to-year variability in wind speed eclipses the log law’s uncertainty in generating error in the annual energy yield.

\(^9\text{[20], p. 44.}\)

\(^10\text{[6], p. 19.}\)
4.1. ENERGY PRODUCTION ESTIMATES

4.1.2 Power Law, $\alpha = 1/7$

Energy Estimate

Using the power law to extrapolate the measured wind speeds from 40 m to 65 m and applying Equation 3.3, we obtain the monthly and annual energy yields for a hypothetical 7-turbine wind farm at Brodie Mountain presented in Table 4.3.

Table 4.3: Monthly and annual estimates for energy production at Brodie Mountain made using the power law. The predicted annual total is 140.2% of Williams College’s 2002-2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>3.209</td>
</tr>
<tr>
<td>Feb</td>
<td>2.309</td>
</tr>
<tr>
<td>Mar</td>
<td>3.097</td>
</tr>
<tr>
<td>Apr</td>
<td>1.813</td>
</tr>
<tr>
<td>May</td>
<td>2.446</td>
</tr>
<tr>
<td>Jun</td>
<td>2.051</td>
</tr>
<tr>
<td>Jul</td>
<td>1.670</td>
</tr>
<tr>
<td>Aug</td>
<td>1.123</td>
</tr>
<tr>
<td>Sep</td>
<td>1.860</td>
</tr>
<tr>
<td>Oct</td>
<td>3.164</td>
</tr>
<tr>
<td>Nov</td>
<td>3.787</td>
</tr>
<tr>
<td>Dec</td>
<td>3.579</td>
</tr>
<tr>
<td>Annual</td>
<td>30.11</td>
</tr>
<tr>
<td>% Wms. Col.</td>
<td>140.2%</td>
</tr>
</tbody>
</table>

The $\alpha = 1/7$ energy estimate is slightly lower than that made using the log law (30.1 vs. 32.1 million kW-hr), though it still represents about 140% of Williams College’s annual electricity use. This should be expected given that the log law always predicts speeds to be slightly higher than does the power law. Additionally, we see the same seasonal behavior in the monthly figures that we observed in the log law predictions. This finding should also be expected since the seasonal variations in energy come from seasonal variations in wind speed that affect both the log- and power-law calculations.

Uncertainty

To estimate the uncertainty in the annual energy production figure, we proceed in much the same manner as in Section 4.1.1. Since the calculations that determine which power curve to use are exactly the same here as above, we simply quote the earlier result: there is not enough uncertainty in temperature, pressure, atmospheric temperature lapse rate, or altitude to have any effect on the final energy figure. However, the uncertainty in the wind-speed measurements made by UMass’ anemometers, as well as the error in the value of $\alpha$, come into play when using the power law to extrapolate wind speeds and
affect the final uncertainty. The partial derivatives of the power law with respect to these variables are:

\[
\frac{\partial U}{\partial U_r} = \left(\frac{z}{z_r}\right)^\alpha \tag{4.10}
\]

\[
\frac{\partial U}{\partial \alpha} = \alpha U_r \left(\frac{z}{z_r}\right)^{\alpha-1} \tag{4.11}
\]

where the reference height \( z_r = 40 \text{ m} \) and the height of the turbine hub \( z = 65 \text{ m} \).

As above, NRG’s #40 Maximum anemometers record wind speeds with a precision of \( \pm 1\% \), so \( \delta U_r \) depends on wind speed. To fix \( \delta \alpha \), we need to determine a reasonable range over which \( \alpha \) could vary. Serendipitously, in calculating how much energy could be produced at the Brodie site using the variable-\( \alpha \) power law (see Section 4.1.3), I came up with just such a range. Over all of 1998, it turns out, \( \alpha \) varied between \(-1.72 \) and \( 3.20 \), with an average value of \( .429 \) (\( \sigma = .34 \)). A reasonable estimate of the uncertainty in \( \alpha \), then, is simply the difference between \( 1/7 \) and the average empirical value. Thus, \( \delta \alpha \approx .43 - 1/7 = .286 \). Combining all this information together using

\[
\delta U = \sqrt{\left(\frac{\partial U}{\partial U_r} \delta U_r\right)^2 + \left(\frac{\partial U}{\partial \alpha} \delta \alpha\right)^2}, \tag{4.12}
\]

we discover that \( .25 \text{ m/s} < |\delta U| < .75 \text{ m/s} \) for \( U_r > 8.5 \text{ m/s} \). This means that data points whose speed is above \( 8.5 \text{ m/s} \) could be offset by at most 1 bin, and that those below \( 8.5 \text{ m/s} \) fall into their respective bins without uncertainty. In contrast with the log law error analysis, in which data points with large errors were at high enough wind speeds not to affect the energy production figure, here the same reasoning does not hold. In order to determine what effect \( \delta U \) has on energy, then, we need to know how much less or more energy would be produced if a certain data point fell into the bin immediately to its left or to its right, respectively. If we take the \( 1.20 \text{ kg/m}^3 \) power curve—which is

Figure 4.5: Monthly power law energy production estimates for a 7-turbine wind farm at Brodie Mountain, 1998.
the steepest and hence has the potential for greatest error—and separately calculate the average difference in power output between every bin and its two nearest neighbors, we can then determine the uncertainty in power produced at each wind speed. That is, if the power output of the turbine is \( P_{i,j} \) Watts for bin \( c_{i-1} \), \( P_{i,j} \) Watts for \( c_i \), and \( P_{i+1,j} \) Watts for \( c_{i+1} \), then the average uncertainty in power output for 10 m/s data intervals is

\[
\delta P_{i,j} = \frac{(P_{i+1,j} - P_{i,j}) + (P_{i,j} - P_{i-1,j})}{2} = \frac{P_{i+1,j} - P_{i-1,j}}{2} \text{ Watts.} \tag{4.13}
\]

If we carry out this calculation for each wind speed between 8.5 m/s and 15 m/s, we can calculate \( \delta E \), the uncertainty in energy yield:

\[
\delta E = \sum_{i=1}^{n} c_i \cdot \delta P_{i,j} \cdot t, \tag{4.14}
\]

where \( i \) is the bin number (if there are \( n \) bins), \( c_i \) is the number of counts in bin \( i \),\(^{11}\) \( \delta P_{i,j} \) is the uncertainty in power for bin \( i \) and power curve \( j \) (calculated according to Equation 4.13), and \( t \) is the time interval (in this case, \( t = 10 \) minutes = 600 seconds). The result of these calculations is that \( \delta E_{Apr} = \pm 122 \) thousand kW-hr, which is 6.8% of the 1.83 million kW-hr produced during that month. Multiplying this value by 12 gives \( \delta E_{year} = \pm 1.47 \) million kW-hr, or about 4.9% of the annual 30.1 million kW-hr.

To summarize, if we assume the power law correctly predicts wind speed, a 7-turbine wind farm on Brodie Mountain in 1998 could have produced \( 30.1 \pm 1.5 \) million kW-hr of electricity.

### 4.1.3 Power Law, Variable \( \alpha \)

#### Energy Estimate

In this section I estimate the energy production at Brodie Mountain in 1998 using the power law with a separate ‘power law exponent’ \( \alpha \) calculated for each time interval. This method extrapolates measured wind speeds upwards to the hub height differently for each 10-minute period. It is hoped that this method will produce a better energy estimate than the ‘one-size-fits-all’ \( \alpha = 1/7 \) used in Section 4.1.2.

To calculate a value for \( \alpha \) given wind speeds at two different heights \( z_1 \) and \( z_2 \), we take the power law

\[
U(z_2) = U(z_1) \cdot (z_2/z_1)^\alpha \tag{4.15}
\]

and solve for \( \alpha \):

\[
\alpha = \frac{\ln(U(z_2)/U(z_1))}{\ln(z_2/z_1)}. \tag{4.16}
\]

The UMass anemometers on Brodie Mountain recorded wind speeds at three different heights above the ground: 10 m, 25 m, and 40 m. Thus, we have the option of calculating \( \alpha \) in three different ways—using \( \{U_{10} \text{ and } U_{40}\} \), \( \{U_{25} \text{ and } U_{40}\} \), or \( \{U_{10} \text{ and } U_{25}\} \). Once we have \( \alpha \), we can use the power law to extrapolate \( U_{40} \) upwards to the hub height at 65 m.

---

\(^{11}\)We use the speed distribution for April 1998 as our ‘representative month’.
4.1. ENERGY PRODUCTION ESTIMATES

Ideally, we would always use $U_{10}$ and $U_{40}$ since the larger the difference in altitudes, the more accurately the power law represents the true wind profile. In practice, though, we cannot always use the most ideal wind speeds, since NRG’s #40 Maximum anemometers have an activation threshold of .78 m/s (1.75 mi/hr). Thus, any wind speed that is below the threshold is inaccurate and cannot be used to calculate $\alpha$. In this way a hierarchy of ‘best-case’ wind speed combinations is established: the first priority should be to use \{$U_{10}$ and $U_{40}$\} because they are most spread out in height. The next best scenario is to use \{$U_{25}$ and $U_{40}$\}, since this combination models the wind profile closer to the hub height (65 m) than \{$U_{10}$ and $U_{25}$\} would. Finally, the least ideal combination is \{$U_{10}$ and $U_{25}$\}. If all three combinations are not viable, either all three anemometers were malfunctioning or wind speeds at all three heights were below the threshold—meaning wind speed was either unknown or low and essentially constant with height. In this case, $U(z_2)$ should be equal to $U(z_1)$, so as a last resort we can assign $\alpha$ to be zero.

If we were to write out this hierarchy in ‘pseudo-code’, it would look something like this:

```plaintext
if( (speed40 > 1.75) and (speed10 > 1.75) )
  { alpha = ln(speed40/speed10) / ln(40/10) }
else if( (speed40 > 1.75) and (speed25 > 1.75) )
  { alpha = ln(speed40/speed25) / ln(40/25) }
else if( (speed25 > 1.75) and (speed10 > 1.75) )
  { alpha = ln(speed25/speed10) / ln(25/10) }
else { alpha = 0 }
```

In order to get Excel to carry out these calculations and produce separate $\alpha$ values for each time interval, the ‘pseudo-code’ needs to be translated into a form Excel can understand. If the wind data in question is in row 1, $U_{10}$ speeds are in column A, $U_{25}$ speeds in column B, and $U_{40}$ speeds in column C, the Excel code to calculate $\alpha$ is:

```excel
IF( AND (C1 > 1.75, A1 > 1.75), (LN(C1/A1) / LN(40/10)), IF( AND (C1 > 1.75, B1 > 1.75), (LN(C1/B1) / LN(40/25)), IF( AND (B1 > 1.75, A1 > 1.75), (LN(B1/A1) / LN(25/10)), 0 ) ) )
```

If we let Excel crunch through the January 1998 wind data, we come up with an average $\alpha$ value of .429 with a standard deviation of .343 ($n = 4461$). The maximum $\alpha$ observed is 3.20 and the minimum is −1.72. Though negative values of $\alpha$ seem strange, they simply mean that for some time intervals the wind speed was faster at lower altitudes. The average value of .429 is quite a bit larger than the standard $\alpha$ of $1/7 = .143$, which suggests that we should get rather different results than we did in the fixed-$\alpha$ case. Running the power law calculations using the variable $\alpha$ values, we obtain the monthly and annual results presented in Table 4.4.

The variable-$\alpha$ energy estimate is quite a bit higher than both the log law and fixed-$\alpha$ estimates (47.6 vs. 32.1 and 30.1 million kW-hr), putting the output of a 7-turbine wind farm around 220% of Williams Colleges annual electricity use. This wildly high estimate is surprising, since we had hoped to get a more reasonable figure using values of $\alpha$ tailor-made for each time interval. In addition, though the seasonal variation in
Table 4.4: Monthly and annual estimates for energy production at Brodie Mountain made using the power law with variable $\alpha$. The predicted annual total is 221.7% of Williams College’s 2002-2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>4.348</td>
</tr>
<tr>
<td>Feb</td>
<td>3.246</td>
</tr>
<tr>
<td>Mar</td>
<td>4.141</td>
</tr>
<tr>
<td>Apr</td>
<td>3.146</td>
</tr>
<tr>
<td>May</td>
<td>3.939</td>
</tr>
<tr>
<td>Jun</td>
<td>3.450</td>
</tr>
<tr>
<td>Jul</td>
<td>3.795</td>
</tr>
<tr>
<td>Aug</td>
<td>2.777</td>
</tr>
<tr>
<td>Sep</td>
<td>3.870</td>
</tr>
<tr>
<td>Oct</td>
<td>4.950</td>
</tr>
<tr>
<td>Nov</td>
<td>5.127</td>
</tr>
<tr>
<td>Dec</td>
<td>4.833</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td></td>
</tr>
<tr>
<td>% Wms. Col.</td>
<td>221.7%</td>
</tr>
</tbody>
</table>

Figure 4.6: Monthly power law (variable $\alpha$) energy production estimates for a 7-turbine wind farm at Brodie Mountain, 1998.
energy production is still observable, it looks less pronounced in this case. It seems, then, that varying $\alpha$ tends to affect lower wind speeds disproportionately more than higher ones. This observation, in and of itself, is essentially enough to discount the results obtained with this method. What we can conclude, then, is that the power law is useful for approximating *average* wind speed with height—but that evidently it is not successful in predicting the *instantaneous* wind speed profile.

**Uncertainty**

Even if we end up discounting the variable-$\alpha$ results, we must still estimate the uncertainty in the annual energy production figure. The analysis required is exactly the same as that used in the previous section concerning the fixed-$\alpha$ power law, except that here the only difference is a slightly higher uncertainty in $\alpha$. Whereas above we decided that $\delta\alpha$ should be the difference between 1/7 and the average value of $\alpha$ (from the variable-$\alpha$ case), here $\delta\alpha$ is simply the standard deviation in $\alpha$. That is, $\delta\alpha = \pm 0.343$. If we proceed as outlined in the previous section using this value of $\delta\alpha$, we find that:

- $0.25 \text{ m/s} < |\delta U| < 0.75 \text{ m/s}$—so data points could be off by $\pm 1$ bin—
  for $2 \text{ m/s} < U_r < 7 \text{ m/s}$,

- $0.75 \text{ m/s} < |\delta U| < 1.25 \text{ m/s}$—so data points could be off by $\pm 2$ bins—
  for $7.5 \text{ m/s} < U_r < 11 \text{ m/s}$, and

- $1.25 \text{ m/s} < |\delta U| < 1.75 \text{ m/s}$—so data points could be off by $\pm 3$ bins—
  for $11.5 \text{ m/s} < U_r < 15 \text{ m/s}$.

We can use this information to calculate the uncertainty in power produced at each wind speed for the 1.20 kg/m$^3$ power curve according to Equations 4.13 and 4.14. We find that the uncertainty in energy for April 1998 is $\delta E_{Apr} = \pm 486$ thousand kW-hr, which is 15.5% of the 3.15 million kW-hr produced during that month. Multiplying by 12 gives $\delta E_{year} = \pm 5.83$ million kW-hr, or about 12.3% of the annual 47.6 million kW-hr.

To summarize, if we assume the variable-$\alpha$ power law correctly predicts wind speed (which it seems not to do), a 7-turbine wind farm on Brodie Mountain in 1998 could have produced $47.6 \pm 5.8$ million kW-hr of energy.

### 4.1.4 Weibull Distribution

**Energy Estimate**

In order to generate a wind speed distribution scaled and binned appropriately for use with GE’s power curves, we must multiply the Weibull distribution (see Section 3.2.1) by the proper scale factor and then chop it up into 0.5 m/s-wide bins. In January, for example, a month with 31 days, there are 4,464 separate 10-minute intervals. If we could get the area under the Weibull curve $p(U)$ to be 4,464—which we can easily do by multiplying $p(U)$ (or $F(U)$) by 4,464—we could then divide up the curve such that $x$ ‘10-minute intervals’ fall between 0 and 0.5 m/s, $y$ ‘10-minute intervals’ fall between 0.5 and 1 m/s, $z$ between 1 and 1.5 m/s, etc., all the way up to 25 m/s. $\Delta F(U)_i$, the
number of ‘intervals’ that fall between \( U_i \) and \( U_{i+1} \) (i.e. the area under \( p(U) \) between \( U_i \) and \( U_{i+1} \)), is

\[
\Delta F(U)_i = B \cdot F(U_{i+1}) - B \cdot F(U_i) = B \cdot \left[ \left( 1 - e^{-(U_{i+1}/c)^k} \right) - \left( 1 - e^{-(U_i/c)^k} \right) \right] \quad (4.17)
\]

\[
= B \cdot \left[ e^{-(U_i/c)^k} - e^{-(U_{i+1}/c)^k} \right], \quad (4.18)
\]

where the scale factor \( B = 4,464 \) (for months with 31 days), site-specific annual \( c = 9.36 \) and \( k = 2.292 \),\(^{12}\) \( i \) is the bin number, and \( U_{i+1} - U_i = 0.5 \text{ m/s} \). Iterating Equation 4.18 over all \( i \) produces a wind speed distribution perfectly tailored to be plugged into the existing spreadsheet infrastructure used in the log and power law estimates. The monthly and annual results of these calculations for a hypothetical 7-turbine wind farm on Brodie Mountain are presented in Table 4.5.

Table 4.5: Monthly and annual estimates for energy production at Brodie Mountain made using the Weibull distribution. The predicted annual total is 184.9% of Williams College’s 2002-2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, ( 10^6 \text{ kW-hr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>3.463</td>
</tr>
<tr>
<td>Feb</td>
<td>3.098</td>
</tr>
<tr>
<td>Mar</td>
<td>3.430</td>
</tr>
<tr>
<td>Apr</td>
<td>3.255</td>
</tr>
<tr>
<td>May</td>
<td>3.329</td>
</tr>
<tr>
<td>Jun</td>
<td>3.222</td>
</tr>
<tr>
<td>Jul</td>
<td>3.295</td>
</tr>
<tr>
<td>Aug</td>
<td>3.329</td>
</tr>
<tr>
<td>Sep</td>
<td>3.222</td>
</tr>
<tr>
<td>Oct</td>
<td>3.363</td>
</tr>
<tr>
<td>Nov</td>
<td>3.287</td>
</tr>
<tr>
<td>Dec</td>
<td>3.430</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td><strong>39.72</strong></td>
</tr>
<tr>
<td>% Wms. Col.</td>
<td><strong>184.9%</strong></td>
</tr>
</tbody>
</table>

Figure 4.7 shows the total energy production by month, while Figure 4.8 is zoomed in to show the small differences between the monthly figures. These graphs look strikingly different from the ones generated with the log and power laws. Instead of a seasonal variation in energy production, we see a small monthly variation that is independent of season and that seems to correlate better with the number of days in each month. This is illustrated most strikingly for February, which we would expect to have one of the highest energy production figures—because of high winds during the winter—but which

\(^{12}\)These values are determined from AWS Truewind’s online New England wind map at the location of the Brodie anemometer tower: 42°36’ N, 73°16’ W.
4.1. ENERGY PRODUCTION ESTIMATES

Figure 4.7: Monthly Weibull distribution energy production estimates for a 7-turbine wind farm at Brodie Mountain, 1998.

Figure 4.8: A close-up of Figure 4.7 to show the small monthly differences in energy production.
instead has the lowest, because it only has 28 days. This anomalous behavior is due to the fact that our \( c \) and \( k \) values represent \textit{annual} averages, and so \textit{of course} we should not expect them to give good monthly estimates—which is exactly what we observe. If, however, we had four separate \textit{seasonal} \( c \) and \( k \) values, (which the New York wind map provides but the New England map does not), we would expect more reasonable monthly predictions. Nevertheless, when we add the inaccurate monthly estimates together, they conspire to produce a quite accurate annual energy prediction.

\textbf{Uncertainty}

To analyze the error in the Weibull result, we proceed in much the same manner as in the previous sections. Once again, since we are using the same air densities as before, there is no error in which power curve we use. There are, however, two other sources of error that we must take into account. The first is an uncertainty in \( c \) and \( k \), which comes from the inherent difficulty in trying to read these values off Truewind’s online maps. That is to say, it is difficult to determine the exact location of the Brodie anemometer tower on the map—and so there is a set of about six points at which the tower could have arguably been located. The standard deviation in the \( c \) and \( k \) values from these six points gives us \( \delta c = \pm 0.251 \) and \( \delta k = \pm 6.32 \times 10^{-4} \) (Table 4.6). The reason we use \( c = 9.36 \) and \( k = 2.292 \) instead of the mean values is because the point 42.601° N, 73.267° W represents the most reasonable location of the tower out of the six possibilities. We use \( \delta c \) and \( \delta k \) to calculate \( \delta p(U) \), the uncertainty in the number of ‘10-minute time intervals’ for each wind speed (i.e. the error in \( c_t \)).

<table>
<thead>
<tr>
<th>Lat, °N</th>
<th>Long, °W</th>
<th>( c )</th>
<th>( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.599</td>
<td>-73.267</td>
<td>9.28</td>
<td>2.292</td>
</tr>
<tr>
<td>42.599</td>
<td>-73.27</td>
<td>9.79</td>
<td>2.292</td>
</tr>
<tr>
<td>42.601</td>
<td>-73.27</td>
<td>9.91</td>
<td>2.292</td>
</tr>
<tr>
<td>42.601</td>
<td>-73.267</td>
<td>9.36</td>
<td>2.292</td>
</tr>
<tr>
<td>42.597</td>
<td>-73.269</td>
<td>9.57</td>
<td>2.293</td>
</tr>
<tr>
<td>42.603</td>
<td>-73.267</td>
<td>9.41</td>
<td>2.291</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td>9.5533</td>
<td>2.292</td>
</tr>
<tr>
<td>std dev</td>
<td></td>
<td>0.2514</td>
<td>6.32 \times 10^{-4}</td>
</tr>
</tbody>
</table>

The partial derivatives of the Weibull distribution with respect to \( c \) and \( k \) are:

\[
\frac{\partial p(U)}{\partial c} = B \frac{U^{k-1} \cdot k^2 \left( c^{-2k} - c^{1-k} \right)}{c^2} e^{-(U/c)^k} 
\]

(4.19)

\[
\frac{\partial p(U)}{\partial k} = B \frac{U^{k-1}}{c^k} \left( 1 + k(k-1)(U/c)^{-1} - k^2(U/c)^{k-1} \right) e^{-(U/c)^k} 
\]

(4.20)
4.1. ENERGY PRODUCTION ESTIMATES

where $B$ is the number of 10-minute intervals in the month. If we calculate

$$\delta p(U) = \sqrt{\left( \frac{\partial p(U)}{\partial c} \delta c \right)^2 + \left( \frac{\partial p(U)}{\partial k} \delta k \right)^2}$$  \hspace{1cm} (4.21)

for each 0.5 m/s increment of $U$ in the range 0 m/s to 25 m/s, we obtain the error in each bin’s counts that is due to the uncertainty in finding the exact location of the anemometer tower on the map.

The second source of error that we must consider is the uncertainty in $c$ and $k$ due not to difficulty in reading the map but instead due to the accuracy of the MesoMap™ modeling system itself. AWS Truewind does not provide uncertainties in $c$ and $k$ per se, though fortunately they do compare MesoMap wind speed predictions with ground measurements at various sites. They determine that the correlation between the their predictions and ground measurements is good to an $R^2$ of .92, and that the standard error of wind speeds reported in the New England wind map is ±0.4 m/s, or 6%\(^{13}\). Thus, wind speeds predicted by the Weibull distribution could be higher or lower by 0.4 m/s, which is to say that the entire Weibull distribution could be shifted up or down by this amount. Since the wind speed frequency distribution bins are 0.5 m/s wide, this means that the counts in each bin could fall into the next-highest or next-lowest bin. If we couple this uncertainty to that in the number of counts in each bin $c_i$ (calculated for each $U_i$, the wind speed of bin $i$, in Equation 4.21), we can determine the maximum and minimum amounts of energy that could possibly be produced using the Weibull distribution. That is, if counts $c_i$ should really be in the next-highest bin $i + 1$, and there is an uncertainty in the counts of ±$\delta p_i$, then the maximum amount of energy that could be produced is

$$E_{\max} = \sum_{i=1}^{n} (c_i + \delta p_i) \cdot P_{i+1,j} \cdot t,$$ \hspace{1cm} (4.22)

where $P_{i+1,j}$ is the power output for bin $i + 1$ and power curve $j$ (we will assume the 1.20 kg/m\(^3\)/s curve: $j = 10$), and $t$ is the time interval (600 seconds). Alternatively, if the counts in bin $i$ should really be in the next-lowest bin $i - 1$, then the minimum amount of energy that could be produced for bin $i$ is

$$E_{\min} = \sum_{i=1}^{n} (c_i - \delta p_i) \cdot P_{i-1,j} \cdot t.$$ \hspace{1cm} (4.23)

Half the difference between the maximum and minimum possible energy values gives us $\delta E$ for one turbine for one month:

$$\delta E = \pm \frac{E_{\max} - E_{\min}}{2}.$$ \hspace{1cm} (4.24)

If we carry through these calculations for April 1998 (our ‘representative’ month), multiply by 7 for the number of turbines and by 12 for the number of months in the

\(^{13}\)Accuracy of the New England wind map estimates: http://truewind.teamcamelot.com/ne/accuracy.html/.
year, we come up with an uncertainty in the annual energy estimate of $9.02 \times 10^6$ kW-hr, which is 22.7% of the 39.7 million kW-hr figure. Percentage-wise, this is the largest error so far. Therefore, assuming that the Weibull distribution correctly predicts wind speed, a 7-turbine wind farm on Brodie Mountain in 1998 could have produced $39.7 \pm 9.0$ million kW-hr of energy.

4.1.5 Rayleigh Distribution

Energy Estimate

Though the Weibull distribution probably predicts energy yield at Brodie Mountain better than the Rayleigh distribution—since we can obtain $c$ and $k$ values from Truewind’s maps—we will estimate energy production using the Rayleigh distribution in order to verify this fact. Though this means of analysis is arguably redundant, it is important to develop it for use in cases where only the mean wind speed $\bar{U}$ is available.

In order to use the Rayleigh distribution, we need a value of the mean wind speed $\bar{U}$ for the site, which can be obtained from Truewind’s online New England map. At 42°36’ N, –73°16’ W and 65 m altitude, the map predicts a value of $\bar{U} = 8.23$ m/s.

$\Delta F(U)_i$, the number of ‘10-minute intervals’ that fall between $U_i$ and $U_{i+1}$, is

$$\Delta F(U)_i = B \cdot F(U_{i+1}) - B \cdot F(U_i) = B \cdot \left[ 1 - e^{-\frac{U_i}{\bar{U}}} \right] - \left[ 1 - e^{-\frac{U_{i+1}}{\bar{U}}} \right]$$

$$= B \cdot \left[ e^{-\frac{U_i}{\bar{U}}} - e^{-\frac{U_{i+1}}{\bar{U}}} \right]$$

where the scale factor $B = 4,464$ is the number of ‘10-minute intervals’ in a month with 31 days, site-specific $\bar{U} = 8.23$, $i$ is the bin number, and $U_{i+1} - U_i = 0.5$ m/s. Carrying through this calculation for each bin produces a wind speed distribution perfectly tailored to be plugged into the existing spreadsheet infrastructure used in the log law, power law, and Weibull estimates. The results of doing so are presented in Table 4.7.

Figure 4.9 shows the total energy production by month, which varies quite similarly to the predictions made with the Weibull distribution (Figure 4.7)—though on average the Rayleigh predictions are 2.5% smaller. This similarity should be expected, of course, because the Rayleigh distribution is just a special case of the Weibull distribution. Here, just as in the previous section, we see that monthly differences seem to correlate well with the number of days in each month instead of varying seasonally as they do in the log and power law cases. Again, this is due to the fact that our $\bar{U}$ value is an annual average; if we had four separate seasonal averages for $\bar{U}$, we would expect better correlation to the actual seasonal variations in wind speed and power production.

Uncertainty

To calculate the error in the Rayleigh prediction, we follow exactly the same methodology as in the Weibull case. Though the most likely location of the Brodie anemometer tower is 42°36’ N, –73°16’ W, the uncertainty of determining the exact location of the wind tower on the map leaves us with several possible alternative values for $\bar{U}$ presented in
Table 4.7: Monthly and annual estimates for energy production at Brodie Mountain made using the Rayleigh distribution. The predicted annual total is 180.3\% of Williams College’s 2002–2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>3.370</td>
</tr>
<tr>
<td>Feb</td>
<td>3.017</td>
</tr>
<tr>
<td>Mar</td>
<td>3.340</td>
</tr>
<tr>
<td>Apr</td>
<td>3.174</td>
</tr>
<tr>
<td>May</td>
<td>3.250</td>
</tr>
<tr>
<td>Jun</td>
<td>3.145</td>
</tr>
<tr>
<td>Jul</td>
<td>3.219</td>
</tr>
<tr>
<td>Aug</td>
<td>3.250</td>
</tr>
<tr>
<td>Sep</td>
<td>3.145</td>
</tr>
<tr>
<td>Oct</td>
<td>3.280</td>
</tr>
<tr>
<td>Nov</td>
<td>3.204</td>
</tr>
<tr>
<td>Dec</td>
<td>3.340</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td></td>
</tr>
<tr>
<td>% Wms. Col.</td>
<td>180.3%</td>
</tr>
</tbody>
</table>

Figure 4.9: Monthly Rayleigh distribution energy production estimates for a 7-turbine wind farm at Brodie Mountain, 1998.
Table 4.8. The error in the mean wind speed is the standard deviation of these values: \( \delta \bar{U} = \pm 0.2211 \text{ m/s} \).

Table 4.8: Values of \( \bar{U} \) at several possible locations of the Brodie anemometer tower.

<table>
<thead>
<tr>
<th>Lat, °N</th>
<th>Long, °W</th>
<th>( \bar{U} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.599</td>
<td>-73.267</td>
<td>8.15</td>
</tr>
<tr>
<td>42.599</td>
<td>-73.27</td>
<td>8.6</td>
</tr>
<tr>
<td>42.601</td>
<td>-73.27</td>
<td>8.71</td>
</tr>
<tr>
<td>42.601</td>
<td>-73.267</td>
<td>8.23</td>
</tr>
<tr>
<td>42.597</td>
<td>-73.269</td>
<td>8.41</td>
</tr>
<tr>
<td>42.603</td>
<td>-73.267</td>
<td>8.27</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td>8.395</td>
</tr>
<tr>
<td>std dev</td>
<td></td>
<td>0.2211</td>
</tr>
</tbody>
</table>

The derivative with respect to \( \bar{U} \) of the Rayleigh distribution is:

\[
\frac{\partial p(U)}{\partial \bar{U}} = \pi B \frac{U}{\bar{U}^3} \left[ \frac{\pi}{4} \left( \frac{U}{\bar{U}} \right)^2 - 1 \right],
\]

where \( B \) is the number of 10-minute intervals in the month. If we calculate

\[
\delta p(U) = \sqrt{\left( \frac{\partial p(U)}{\partial \bar{U}} \delta \bar{U} \right)^2} = \frac{\partial p(U)}{\partial \bar{U}} \delta \bar{U}
\]

for each 0.5 m/s increment of \( U \) in the range 0 m/s to 25 m/s, we get the error in each bin’s counts that is due to the uncertainty in finding the exact location of the anemometer tower on the map. If we then consider the \( \pm 0.4 \text{ m/s} \) error in MesoMap’s speed predictions, and calculate the maximum and minimum energy production values according to Equations 4.22 and 4.23, we end up with an uncertainty \( \delta E = \pm 7.4 \times 10^6 \text{ kW-hr} \), which is 19.2\% of the 38.7 million kW-hr annual figure. This error, though slightly smaller than that in the Weibull prediction, is still fairly large compared to those in the log and power law estimates. Therefore, a 7-turbine wind farm on Brodie Mountain could have produced 38.7 \pm 7.4 million kW-hr of energy in 1998, assuming that the Rayleigh distribution correctly predicts wind speed.

### 4.1.6 The Local Wind Resource

In this section, we calculate the total amount of energy available to an ideal turbine at Brodie Mountain in 1998. To do so, we use the ‘theoretical maximum power curve’ (see Equation 3.1):

\[
P(U) = \frac{1}{2} \cdot \frac{16}{27} \rho A U^3,
\]

(4.29)
where \( \rho \) is the air density (which varies by month), \( A \) is the area swept out by the rotor blades (blade diameter = 70.5 m so \( A = 3904 \text{ m}^2 \)), and \( U \) is the wind speed.

To calculate the energy available each month, we do so in essentially the same way as in the log law case—only here, we do not select one of GE’s prefabricated power curves. Instead, we create our own using Equation 4.29 to determine the power available for winds with speeds at the center of each bin. That is, we calculate \( P(0 \text{ m/s}), P(.5 \text{ m/s}), P(1 \text{ m/s}), \) etc.; the results are presented in Table 4.9.

Table 4.9: The wind resource at Brodie Mountain in 1998. The predicted annual total is 312.2\% of Williams College’s 2002–2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, ( 10^6 \text{ kW-hr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>7.158</td>
</tr>
<tr>
<td>Feb</td>
<td>5.383</td>
</tr>
<tr>
<td>Mar</td>
<td>7.358</td>
</tr>
<tr>
<td>Apr</td>
<td>3.264</td>
</tr>
<tr>
<td>May</td>
<td>4.898</td>
</tr>
<tr>
<td>Jun</td>
<td>4.965</td>
</tr>
<tr>
<td>Jul</td>
<td>2.804</td>
</tr>
<tr>
<td>Aug</td>
<td>1.884</td>
</tr>
<tr>
<td>Sep</td>
<td>3.576</td>
</tr>
<tr>
<td>Oct</td>
<td>6.517</td>
</tr>
<tr>
<td>Nov</td>
<td>9.504</td>
</tr>
<tr>
<td>Dec</td>
<td>9.743</td>
</tr>
<tr>
<td>Annual</td>
<td>67.05</td>
</tr>
<tr>
<td>% Wms. Col.</td>
<td>312.2%</td>
</tr>
</tbody>
</table>

As we expect, the prediction of an annual 67.0 million kW-hr, which is about 312\% of Williams College’s yearly energy use, is more than a factor of 2 greater than, e.g., the log law prediction of 32.1 million kW-hr. This result signifies that typical modern turbines can extract only about 50\% of the available energy in the wind.

4.2 Comparison of the Six Energy Estimation Methods

Now that we have obtained annual energy predictions using all six methods outlined in Section 4.1, we can compare them and come up with best candidates for predicting energy yield with an anemometer-data method (log, \( \alpha = 1/7 \) power, or variable-\( \alpha \) power law) and a Truewind method (Weibull or Rayleigh). Doing so will allow us to make equally accurate energy estimates using only two—instead of five—methods. Additionally, comparing the best Truewind and surface-data methods will allow us to determine how accurately Truewind can predict energy yield in the absence of anemometer data. Figures 4.11–4.13 provide the basis for this comparison.
4.2. COMPARISON OF THE SIX ENERGY ESTIMATION METHODS

Figure 4.10: The monthly wind resource at Brodie Mountain, 1998.

Figure 4.11: Dependence of predicted energy yield on prediction method, arranged by month (7-turbine wind farm at Brodie Mountain, 1998).
4.2. COMPARISON OF THE SIX ENERGY ESTIMATION METHODS

Figure 4.12: Dependence of predicted energy yield on prediction method, arranged by method (7-turbine wind farm at Brodie Mountain, 1998).

Figure 4.13: Dependence of annual energy yield on prediction method, with error bars (7-turbine wind farm at Brodie Mountain, 1998).
4.2. COMPARISON OF THE SIX ENERGY ESTIMATION METHODS

The variable-α power law predicts the annual energy production to be rather high. I was inspired to invent this method of velocity extrapolation because of Manwell et al.’s warning that “in practice, the exponent α is a highly variable quantity” (p. 44). My hope was that by calculating a separate value of α for each time interval, we would be able to model the changing wind profile in ‘real time’, and hence end up with a more reasonable estimate of energy than could be produced by using the static value of 1/7. However, it seems that the power law is better suited to extrapolate average—rather than instantaneous—wind speed since the variable-α prediction is higher than both of the Truewind predictions, which we know empirically to be reasonably precise (they model winds to within ±0.4 m/s). Also, the variable-α prediction is higher than both the log and fixed-α predictions, which have also been repeatedly tested and found empirically to provide reasonable estimates. Thus, if we are going to use the power law at all, the fixed-α method seems to be more reliable than the variable-α method.

We can next compare the fixed-α power law to the log law. Both methods provide similar estimates of energy production (30.1 vs. 32.1 million kW-hr), though the better option is the log law. Whereas this extrapolation rule is calculated from first principles, the power law is empirical and so is not as firmly rooted in physical theory. Also, given the dominant but unknown uncertainties in the power curves $P_{i,j}$, the uncertainty in the power law estimate is larger than that in its logarithmic counterpart. Thus, the optimal anemometer-data prediction technique is the log law method.

To within the uncertainty inherent in the predictions, the Weibull and Rayleigh estimates are identical. However, given that we would like to choose one, the Weibull distribution is the better option. The reason is because it can more accurately model the actual wind speed distribution since it has two parameters to the Rayleigh’s one. Though at Brodie, apparently, these two distributions are similar enough that the two distributions are not significantly different, we can imagine a site with a strange wind regime where the two models would not be equivalent. Therefore, the optimal Truewind prediction technique is the Weibull method.

4.2.1 Truewind Accuracy

We can now finally compare the log law prediction to the Weibull prediction in order to decide whether it is possible to make accurate energy forecasts with Truewind’s maps in the absence of on-site anemometer data. If we take the amount of energy predicted by the log law as the ‘true’ value of energy that could have been produced at Brodie Mountain in 1998, we obtain the monthly and annual percentage differences presented in Table 4.10.

Though the monthly Weibull figures range from as high as 168% above (August 1998) to 17.4% below (November 1998) the log law figures, they even out to an annual difference of only 23.8%. If we add in the uncertainty in the Weibull estimate (±9.0 million kW-hr), the annual percentage difference could be anywhere from −4% to 52%. In cases where we have seasonal averages for $c$ and $k$, Truewind should predict monthly energy production reasonably well—though here, since we only have annual parameter values,
Table 4.10: Monthly and annual percentage differences between log law and Truewind Weibull predictions. Brodie Mountain 1998 (7 turbines).

<table>
<thead>
<tr>
<th>Month</th>
<th>TW % Diff from LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>3.0%</td>
</tr>
<tr>
<td>Feb</td>
<td>26.3%</td>
</tr>
<tr>
<td>Mar</td>
<td>5.1%</td>
</tr>
<tr>
<td>Apr</td>
<td>65.6%</td>
</tr>
<tr>
<td>May</td>
<td>26.6%</td>
</tr>
<tr>
<td>Jun</td>
<td>47.9%</td>
</tr>
<tr>
<td>Jul</td>
<td>78.4%</td>
</tr>
<tr>
<td>Aug</td>
<td>168.2%</td>
</tr>
<tr>
<td>Sep</td>
<td>58.9%</td>
</tr>
<tr>
<td>Oct</td>
<td>−0.2%</td>
</tr>
<tr>
<td>Nov</td>
<td>−17.3%</td>
</tr>
<tr>
<td>Dec</td>
<td>−9.0%</td>
</tr>
<tr>
<td>Annual</td>
<td>23.8%</td>
</tr>
</tbody>
</table>

The monthly Truewind predictions are not particularly useful. In addition, since wind speed distributions on site vary from year to year, this annual percentage difference could potentially be larger or smaller depending on whether the year in question is either more or less windy than 1998 (see Section 4.3). We can conclude that estimates made using Truewind’s Weibull distributions are fairly accurate—to within 20% or thereabouts—so we can in fact use them to get a good estimate of a site’s energy production capacity. Over the course of Chapters 5–7 we shall refine this estimate to determine the average accuracy of Truewind predictions.

4.3 Comparison of 1997 to 1998 Log Law Predictions

To obtain a broader picture of the wind’s behavior at Brodie Mountain, it is important to estimate the annual variation in energy production. Table 4.11 and Figure 4.14 compare monthly and annual log-law energy estimates for UMass’ 1997 and 1998 Brodie Mountain data sets.

Over the course of the year, the monthly figures average out to an annual difference of about 9.6%. Because we have considered only two years’ worth of data, however, we cannot be certain that annual energy yield typically fluctuates by such a small amount. While we do in fact have additional Brodie data from 1996 and 1999, neither of these data sets is complete. Still, it would be possible to compare what additional data we do have to the present estimates in order to determine a mean and standard deviation for
4.3. COMPARISON OF 1997 TO 1998 LOG LAW PREDICTIONS

Table 4.11: Monthly and annual percentage differences between 1997 and 1998 log-law predictions at Brodie Mountain (7 turbines).

<table>
<thead>
<tr>
<th>Month</th>
<th>1997 Log Law, $10^6$ kW-hr</th>
<th>1998 Log Law, $10^6$ kW-hr</th>
<th>1997 % Diff from 1998</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>3.361</td>
<td>4.514</td>
<td>34.3%</td>
</tr>
<tr>
<td>Feb</td>
<td>2.454</td>
<td>3.558</td>
<td>45.0%</td>
</tr>
<tr>
<td>Mar</td>
<td>3.265</td>
<td>4.179</td>
<td>28.0%</td>
</tr>
<tr>
<td>Apr</td>
<td>1.966</td>
<td>2.860</td>
<td>45.5%</td>
</tr>
<tr>
<td>May</td>
<td>2.630</td>
<td>3.553</td>
<td>35.1%</td>
</tr>
<tr>
<td>Jun</td>
<td>2.179</td>
<td>1.862</td>
<td>−14.5%</td>
</tr>
<tr>
<td>Jul</td>
<td>1.846</td>
<td>1.928</td>
<td>4.4%</td>
</tr>
<tr>
<td>Aug</td>
<td>1.241</td>
<td>1.520</td>
<td>22.4%</td>
</tr>
<tr>
<td>Sep</td>
<td>2.028</td>
<td>2.431</td>
<td>19.8%</td>
</tr>
<tr>
<td>Oct</td>
<td>3.369</td>
<td>2.424</td>
<td>−28.0%</td>
</tr>
<tr>
<td>Nov</td>
<td>3.977</td>
<td>3.277</td>
<td>−17.6%</td>
</tr>
<tr>
<td>Dec</td>
<td>3.769</td>
<td>3.057</td>
<td>−18.9%</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td><strong>32.09</strong></td>
<td><strong>35.16</strong></td>
<td><strong>9.6%</strong></td>
</tr>
</tbody>
</table>

Figure 4.14: Comparison of 1997 and 1998 log-law energy predictions for Brodie Mountain (7 turbines).
yearly energy production. In any case, without performing these calculations we are able to conclude that the annual variation in energy production is around ±10%.

4.4 Wind Direction at Brodie Mountain

While the speed distribution of winds at the site tells us how much energy could be generated, the directional distribution helps us determine whether or not the topography would allow us to actually collect that energy. If the prevailing wind direction at a linear ridge is along the ridgeline, for example, most of the turbines built along it are ‘shaded’ by their upwind neighbors. In this scenario, downwind turbines are not subjected to the full force of the wind as it flows through the site, since some of the energy is ‘used up’ by the upwind ones—and so the farm does not produce energy at the expected rate. On the other hand, if the prevailing wind is orthogonal to that same ridge, then no turbine is shaded by any other and each produces the maximum amount of energy. Thus, the relation of the speed distribution to the local topography is important in assessing the feasibility of any proposed wind project.

Figure 4.15: Directional distribution of winds at Brodie Mountain in 1998, as measured by RERL’s 40 m wind vane. The bars represent the percentage of time the wind blew in each 22.5° sector over the course of the year. 0° is due north.

Figure 4.15, which is a radial histogram created using Microcal’s Origin software,

15I leave this comparison to a future student.
shows the annual directional distribution—sometimes called a ‘wind rose diagram’—of the winds at the Brodie Mountain site. We can see that about 20% of the wind blows out of the WNW, with some 50–60% emanating from between the SW and N. Since the Brodie Mountain ridge line runs approximately N–S (see Figure 4.1), the predominant wind direction is favorable because it means that the prevailing winds blow orthogonally to the ridge. As there may not be significant shading problems, the Brodie site’s topography seems conducive to supporting a wind farm.

Just as we wanted to know how the Truewind energy prediction compared to the log law prediction—so we could determine whether it would be possible to estimate energy production without actual surface data—so too do would we like to know how the Truewind rose diagram compares to the measured directional distribution. The annual Truewind directional distribution for the Brodie site (42°36' N, −73°16' W), which is available on AWS Truewind’s website, is presented in Figure 4.16.

![Wind Rose Diagram](image)

Figure 4.16: Directional distribution of winds at Brodie Mountain (42°36' N, −73°16' W). Light bars represent the percentage of time the wind blew in each 22.5° sector over the course of the year, while the dark bars represent the percentage of total energy that would be generated by winds from each sector. Courtesy AWS Truewind LLC.

The predicted directional distribution (Figure 4.16) is in remarkable agreement with

---

the data-derived distribution (Figure 4.15). It correctly predicts that the greatest
amount of wind—about 20%—blows from the WNW, and that the majority of the wind
emanates from between the SSW and N. However, it seems to slightly overestimate the
SSW-erly component; whereas Truewind predicts roughly 14% of the wind to come from
the SSW, the data show that particular component originating more from the SW and
representing only about 7% of the total wind. Despite these small differences, however,
the Truewind distribution fairly accurately models the site data. This result suggests
that we are probably justified in using Truewind’s rose diagrams to obtain a good first
estimate of the directional distribution at a given site. If we would like to obtain a more
accurate directional distribution, however, we would need to install wind vanes there.

4.5 Summary of Results

In this chapter, we predicted the energy production for a 7-turbine wind farm at Brodie
Mountain (1998) in five separate ways: three whose speed distributions were derived
from on-site anemometer data (log law, fixed-\( \alpha \) power law, variable-\( \alpha \) power law) and
two whose speed distributions were derived from AWS Truewind’s predictions (Weibull
distribution and Rayleigh distribution). We also calculated the total wind resource at
the site and determined the uncertainty in each prediction. The results are summarized
in Table 4.12.

Table 4.12: Summary of annual energy production estimates and errors for all six pre-

<table>
<thead>
<tr>
<th></th>
<th>Log Law, ( z_0 = .5 ) m</th>
<th>Power Law, ( \alpha = 1/7 )</th>
<th>Power Law, ( \alpha ) variable</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annual Yield, 10^6 kW-hr</strong></td>
<td>32.09</td>
<td>30.11</td>
<td>47.62</td>
</tr>
<tr>
<td>Error, 10^6 kW-hr</td>
<td>0.00</td>
<td>1.47</td>
<td>5.83</td>
</tr>
<tr>
<td>% Error</td>
<td>0.0%</td>
<td>4.9%</td>
<td>12.3%</td>
</tr>
<tr>
<td></td>
<td>Weibull</td>
<td>Rayleigh</td>
<td>Local Wind Resource</td>
</tr>
<tr>
<td><strong>Annual Yield, 10^6 kW-hr</strong></td>
<td>39.72</td>
<td>38.74</td>
<td>67.05</td>
</tr>
<tr>
<td>Error, 10^6 kW-hr</td>
<td>9.02</td>
<td>7.44</td>
<td>—</td>
</tr>
<tr>
<td>% Error</td>
<td>22.7%</td>
<td>19.2%</td>
<td>—</td>
</tr>
</tbody>
</table>

Next, we compared these results and determined that the best anemometer-data
method for predicting energy is the log law technique, while the best Truewind method
is the Weibull distribution technique. We found that, for 1998, the Weibull distribution
predicts about 24% more energy production at the site than the log law does, but that
the Weibull distribution provides a good first estimate of annual energy production in
the absence of on-site anemometer data. By comparing UMass’ 1997 and 1998 data
sets, we also concluded that there is an approximate variation in year-to-year energy
production of ±10%. In addition, we determined that the prevailing winds at the Brodie
4.5. SUMMARY OF RESULTS

Site emanate from the WNW, which is conducive to supporting a wind farm since the site’s ridgeline runs approximately N–S. Finally, we noted that Truewind’s predicted directional distribution agreed well with the measured distribution, and so their wind rose diagrams can provide a good first estimate of directional distribution. Thus, at sites where we lack raw wind data (such as Berlin Pass), we can be confident that we will obtain reasonable results when we use AWS Truewind’s Weibull distributions and wind rose diagrams to estimate energy production and directional wind distribution.
Chapter 5

Lincoln Labs Data Analysis

Between June and November 2001, a group of MIT Lincoln Laboratory researchers (including Drs. Paul E. Bieringer, David A. Clark, and Michael P. Matthews) maintained a network of weather monitoring stations in the northwestern corner of Massachusetts ([9], [1]). The purpose of this network, dubbed the “Berkshire Mesonet”, was to monitor weather conditions in order to learn about weather forecasting in an area with complex, mountainous terrain. Since the U.S. has undertaken military operations in topographically similar areas (e.g. Kosovo), it was hoped that understanding how better to predict the weather in these regions would contribute to the success of future such operations.

Among the various instruments mounted on the Berkshire Mesonet towers were anemometers and wind vanes. These instruments, deployed at nine sites in and around Williamstown, MA (Table 5.1 and Figure 5.1) provide another proxy—in addition to UMass’ data from Brodie Mountain—for estimating the wind energy that could be produced at Berlin Pass. At each site, the anemometers were mounted 10 m above the ground (open locations) or 10 m above the canopy (∼16.1 m above the ground; wooded locations).

The Lincoln Lab researchers error-corrected and reformatted the raw Mesonet data to be read and displayed in an application called GrADS. GrADS (Grid Analysis and Display System) is an “interactive desktop tool that is used for easy access, manipulation, and visualization of earth science data” developed by researchers at the Center for Ocean-Land-Atmosphere Studies (COLA) and funded by NASA, NSF, and NOAA. To convert the data from its GrADS format into a tab-delimited text file, which I could then be paste into an Excel spreadsheet for analysis, I wrote a short program called WindData.gs to do so (see Appendix B for code and sample output).

5.1 Energy Production Estimates

In Chapter 4 we concluded that the log law and Weibull distribution methods provide the best predictions of energy yield, so we will consider only these two models here.

5.1. ENERGY PRODUCTION ESTIMATES

Table 5.1: Site characteristics of the Berkshire Mesonet weather towers (Bieringer (2003), unpublished appendix). Wind data measured approximately 10 m above the canopy (or ground, in the case of open sites).

<table>
<thead>
<tr>
<th>Location</th>
<th>Site ID</th>
<th>Lat, °N</th>
<th>Lon, °W</th>
<th>Elev, m</th>
<th>Vegetation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farm Site</td>
<td>FRM</td>
<td>42.673</td>
<td>-73.196</td>
<td>323</td>
<td>Open</td>
</tr>
<tr>
<td>Mt. Greylock Summit</td>
<td>GLH</td>
<td>42.637</td>
<td>-73.168</td>
<td>1064</td>
<td>Wooded</td>
</tr>
<tr>
<td>Harriman-West Airport</td>
<td>HWA</td>
<td>42.697</td>
<td>-73.163</td>
<td>199</td>
<td>Open</td>
</tr>
<tr>
<td>Mt. Rainer Summit</td>
<td>MTR</td>
<td>42.716</td>
<td>-73.283</td>
<td>781</td>
<td>Wooded</td>
</tr>
<tr>
<td>Notch Road Site</td>
<td>NCH</td>
<td>42.671</td>
<td>-73.156</td>
<td>530</td>
<td>Wooded</td>
</tr>
<tr>
<td>Greylock School Site</td>
<td>SCH</td>
<td>42.674</td>
<td>-73.235</td>
<td>287</td>
<td>Open</td>
</tr>
<tr>
<td>Taconic Ridge</td>
<td>TCN</td>
<td>42.712</td>
<td>-73.249</td>
<td>469</td>
<td>Open</td>
</tr>
<tr>
<td>Williamstown Landfill</td>
<td>WLL</td>
<td>42.732</td>
<td>-73.209</td>
<td>186</td>
<td>Open</td>
</tr>
<tr>
<td>W’town Water Tower</td>
<td>WTR</td>
<td>42.718</td>
<td>-73.177</td>
<td>240</td>
<td>Wooded</td>
</tr>
</tbody>
</table>

Figure 5.1: Terrain elevation (in m a.s.l.) in the region of the Berkshire Mesonet (western MA, southern VT, eastern NY). The circles mark the locations of the surface observation towers. Courtesy of Dr. Paul Bieringer, MIT Lincoln Labs (Bieringer (2003)).
5.1.1 Log Law: MTR

The Berkshire Mesonet site closest and physically most similar to the Berlin Pass site is the summit of Mount Rainer (site MTR; 42.716° N, −73.283° W; Berlin, NY). If we run WindData.gs on the MTR data and plug the resulting files into an Excel spreadsheet, we obtain the results presented in Table 6.2.

Table 5.2: Monthly and annual log law estimates of energy production for 7 turbines at Mt. Rainer (2001). The predicted annual total is 174.7% of Williams College’s 2002-2003 energy use (excluding the September data, of which ∼96% is missing).

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, 10^6 kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jun</td>
<td>1.979</td>
</tr>
<tr>
<td>Jul</td>
<td>1.636</td>
</tr>
<tr>
<td>Aug</td>
<td>1.799</td>
</tr>
<tr>
<td>Sep</td>
<td>1.005</td>
</tr>
<tr>
<td>Oct</td>
<td>4.643</td>
</tr>
<tr>
<td>Nov</td>
<td>4.693</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td><strong>15.76</strong></td>
</tr>
<tr>
<td>Annual</td>
<td>34.53</td>
</tr>
<tr>
<td>% Wms Col</td>
<td>160.7%</td>
</tr>
<tr>
<td><strong>Annual w/o Sep</strong></td>
<td><strong>37.53</strong></td>
</tr>
<tr>
<td>% Wms Col</td>
<td>174.7%</td>
</tr>
</tbody>
</table>

The most obvious difference between these results and those obtained for Brodie Mountain is that here we only have six monthly estimates. This difference is of course due to the fact that the Berkshire Mesonet only recorded data for a six-month period. To obtain an estimate of annual energy production at Mt. Rainer, then, I calculated that at Brodie Mountain in 1998 the energy production between June and November made up approximately 45.6% of the annual total. Assuming that the same ratio holds here, all we need to do is to add up the monthly energy figures for Mt. Rainer (they sum to 1.58 × 10^7 kW-hr) and divide by .456 to estimate the energy that could have been produced over the whole course of 2001: 34.5 million kW-hr, or 160.7% of Williams College’s 2002-2003 energy consumption. This approximation is reasonable, since Mt. Rainer and Brodie Mountain are in geographically similar areas, are fairly close to each other (roughly 25 km apart), and have similar wind regimes (compare Figure 5.7 to Figure 4.16).

Another difference between the graph in Figure 5.2 and its Brodie analog is that energy production at Mt. Rainer in September 2001 is too low because ∼96% of that month’s data are missing. If we ignore the September data, we can calculate the annual energy production from the remaining five months. At Brodie Mountain in 1998, then, the months June–August and October–November made up approximately 39.3% of the yearly total. If we add up the energy for these five months and then divide by .393, we get an annual figure of 37.5 million kW-hr, or 174.7% of Williams’ annual energy use.
5.1. ENERGY PRODUCTION ESTIMATES

Figure 5.2: Monthly log law energy production estimates for 7 turbines at Mt. Rainer (2001).

This figure represents the best estimate of the annual energy production at Mt. Rainer that we can make given the incompleteness of the data set. It is difficult to say whether it represents an underestimate or an overestimate of the energy, since we do not have data from the winter, which is the windiest time of the year.

The prediction of 37.5 million kW-hr for Mt. Rainer in 2001 is about 17% greater than that for Brodie Mountain in 1998 (32.1 million kW-hr). Although we know from the previous chapter that there is at least a 10% variation in energy output from year to year, this difference is probably large enough to be significant despite the statistical variation. The cause of this energy difference between the sites cannot be explained by their difference in elevation—Mt. Rainer peaks at 781 m above sea level, while Brodie Mountain levels off at approximately 790 m. A more plausible explanation is simply that Mt. Rainer is subject to higher winds than is Brodie Mountain.

The error in the annual estimates, calculated according to the method laid out in Section 4.1.1, is only ±945 kW-hr. The reason it is not zero, as it is for the Brodie Mountain data, is because the MTR anemometer recorded wind speeds at a height of 16.1 m above the ground. Since the speeds need to be extrapolated upwards from 16.1 m to 65 m (as opposed to the extrapolation from 40 m to 65 m for the Brodie data), there is more room for error to sneak in. Even so, the uncertainty in wind speed is only large enough for data points to be placed in the wrong bin if they are at or above 14 m/s. Since GE’s power curves become flat at 14.5 m/s, only frequency counts at exactly 14 m/s can cause any uncertainty in the final energy estimate. In November 2001, which we shall use as a ‘representative’ month, there were only 90 five-minute intervals during which the wind speed averaged 14 m/s. And since the uncertainty in power is only 1.5 kJ per five-minute interval for winds of 14 m/s, the total annual uncertainty for seven turbines comes out to the tiny figure of 945 kW-hr. Therefore, a 7-turbine wind farm on Mt. Rainer in 2001 (a physical impossibility, since there would be room for at most one turbine at the summit) could have produced 37.5 ± 0.001 million kW-hr of energy.
5.1.2 Weibull Distribution: MTR

AWS Truewind’s New York wind map provides seasonal values for the Weibull distribution’s $c$ (scale) and $k$ (shape) parameters with which we can model the wind speed distributions at Mt. Rainer as a check on our log law estimates. I had some difficulty in determining the exact location of the site on Truewind’s map, however. Whereas [1] reports the MTR wind tower to be at $42.716^\circ$ N, $-73.283^\circ$ W, Truewind places the summit of Mt. Rainer at approximately $42.715^\circ$ N, $-73.296^\circ$ W. As to why the longitude coordinates are so different we can only speculate: either Truewind’s map is not ‘georeferenced’ correctly or Bieringer et al. did not measure the site’s coordinates correctly, or both. Since we must read the values off Truewind’s map, however, the best strategy is to locate the summit of Mt. Rainer on that map and record the $c$ and $k$ values from that point, regardless of what lat-lon coordinate is reported. In this way we ensure that we have values from as close to the tower location as possible, regardless of whose coordinate system is incorrect. For site MTR, we obtain the values presented in Table 5.3.

Table 5.3: Seasonal $c$ and $k$ values for the Weibull wind speed distribution at Mt. Rainer ($42.716^\circ$ N, $-73.283^\circ$ W; Berlin, NY) provided by AWS Truewind’s New York state wind map.

<table>
<thead>
<tr>
<th>Season</th>
<th>$c$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>9.928</td>
<td>2.229</td>
</tr>
<tr>
<td>Summer</td>
<td>7.86</td>
<td>2.186</td>
</tr>
<tr>
<td>Fall</td>
<td>10.145</td>
<td>2.279</td>
</tr>
<tr>
<td>Winter</td>
<td>12.036</td>
<td>2.481</td>
</tr>
</tbody>
</table>

If we generate wind speed distributions using these parameters (spring: Mar, Apr, May; summer: Jun, Jul, Aug; fall: Sep, Oct, Nov; winter: Dec, Jan, Feb) and calculate the annual energy production according to the method laid out in the previous chapter, we obtain the results presented in Table 5.4.

The plot presented in Figure 5.3 contrasts with Fig 4.5 in the previous chapter because we see a distinct seasonal variation in energy production estimates here whereas in the Brodie estimates we do not. This difference is of course due to the fact that AWS Truewind’s New England wind map provides only annual averages for $c$ and $k$, while the New York map provides a separate pair for each season. We must note, however, that the variation between the monthly estimates for each season (e.g. March, April, and May, etc.) only depends on which power curve was used (which is determined from calculated air density differences). Because of this, the individual monthly estimates are certainly better than those calculated with annual $c$ and $k$ averages (e.g. Brodie Weibull estimates)—though they are probably not as good as those obtained from actual wind data. In addition, the yearly total for Mt. Rainer is probably a slightly more accurate figure than that calculated with the Weibull distribution for Brodie Mountain because of the enhanced seasonal resolution in $c$ and $k$. To quantify this increased accuracy is
5.1. ENERGY PRODUCTION ESTIMATES

Table 5.4: Monthly and annual Truewind (Weibull) estimates of energy production for 7 turbines at Mt. Raimer (2001). Williams College’s 2002-2003 energy use is 53.2% of the predicted annual total.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>4.943</td>
</tr>
<tr>
<td>Feb</td>
<td>4.465</td>
</tr>
<tr>
<td>Mar</td>
<td>3.749</td>
</tr>
<tr>
<td>Apr</td>
<td>3.597</td>
</tr>
<tr>
<td>May</td>
<td>3.683</td>
</tr>
<tr>
<td>Jun</td>
<td>2.298</td>
</tr>
<tr>
<td>Jul</td>
<td>2.375</td>
</tr>
<tr>
<td>Aug</td>
<td>2.375</td>
</tr>
<tr>
<td>Sep</td>
<td>3.663</td>
</tr>
<tr>
<td>Oct</td>
<td>3.818</td>
</tr>
<tr>
<td>Nov</td>
<td>4.913</td>
</tr>
<tr>
<td>Dec</td>
<td>4.361</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td><strong>43.61</strong></td>
</tr>
<tr>
<td>% Wms Col</td>
<td><strong>203.0%</strong></td>
</tr>
</tbody>
</table>

Figure 5.3: Monthly Truewind (Weibull) energy yield estimates for 7 turbines at Mt. Raimer (2001).
5.1. ENERGY PRODUCTION ESTIMATES

difficult, however, since we do not have a full year’s worth of Mt. Raimer data with which to compare the Weibull prediction. Still, we will compare the Weibull and log law predictions—as soon as we calculate the uncertainty in the annual Weibull estimate.

To do so, we proceed in essentially the same manner outlined in the previous chapter. The uncertainties in the $c$ and $k$ values that are due to the difficulty of exactly locating the site of the Mt. Raimer measurement tower on Truewind’s map are approximately $\delta c = 0.032$ and $\delta k = 2.5 \times 10^{-3}$. Assuming an error in Truewind’s predicted wind speeds of $\pm 0.4 \text{ m/s}$, and carrying out the specified calculations for the representative month of April, we obtain an annual uncertainty in energy production of $\pm 8.30 \times 10^6 \text{ kW-hr}$, or $\pm 19.0\%$ of the annual value.

This error has approximately the same magnitude as the Brodie Weibull estimate (22.7%) because of two offsetting errors. First, it is easier to locate the summit of Mt. Raimer on Truewind’s maps than it is to locate the site of the Brodie anemometer tower. This is because the only description I could find of the Brodie site was a lat-lon coordinate; I was unable to determine if the tower was on the crest of the ridge or slightly down slope. Thus, the range of possible locations of the Brodie tower is larger than for the Mt. Raimer tower, leading to a larger error in the Brodie $c$ (scale) value ($\delta c = \pm 0.251$ for Brodie vs. $\delta c = \pm 0.032$ for MTR). However, the shape of the wind speed distributions at the many possible Brodie locations are more similar to each other than those at the several Mt. Raimer locations, so the error in Brodie’s $k$ (shape) parameter is smaller than Raimer’s ($\delta k = \pm 6.32 \times 10^{-4}$ for Brodie vs. $\delta k = \pm 2.5 \times 10^{-3}$ for MTR). These errors offset each other, apparently, and result in the total error being about the same in both cases. Thus, a 7-turbine wind farm on Mt. Raimer in 2001 could have produced $43.6 \pm 8.3 \text{ million kW-hr}$ of electricity.

5.1.3 Truewind Accuracy

We are now in a position to compare the log law and Weibull predictions for Mt. Raimer. If we take the log law estimates as the “true” amounts of energy that could be produced for each month at Mt. Raimer in 1998, we obtain the monthly and annual percentage differences presented in Table 5.5.

The maximum monthly percentage difference of 251%, which occurs in September (Table 5.5), is significantly higher than the other values. This anomaly is of course due to the fact that 96% of the September anemometer data are missing, so we can safely ignore that month altogether. Otherwise, the percentages only vary between 45% (July) to $-21\%$ (November). This range is markedly smaller than the same percentage differences calculated for Brodie Mountain in the previous chapter (168% to $-17\%$). This smaller range of differences is attributable to two factors: first, we only have five months’ worth of valid anemometer data, so it is possible that we would see greater variation given several (or seven) more months’ wind data; second, it is due to the fact that these estimates were made with seasonal $c$ and $k$ values instead of annual ones. Thus, this result verifies our earlier hunch—that seasonal $c$ and $k$ values are better for making monthly predictions of energy production than are annual averages.

But what about yearly energy estimates? How do seasonal $c$ and $k$ values do in comparison to annual ones over the course of 12 months? The yearly percentage difference
Table 5.5: Monthly and annual percentage differences between the log law and Truewind Weibull predictions for 7 turbines at Mt. Raimer, 2001. *The September data may be ignored.

<table>
<thead>
<tr>
<th></th>
<th>Log Law, $10^6$ kW-hr</th>
<th>TW Weibull, $10^6$ kW-hr</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>1.979</td>
<td>4.943</td>
<td>16.1%</td>
</tr>
<tr>
<td>Feb</td>
<td>1.636</td>
<td>4.465</td>
<td>45.1%</td>
</tr>
<tr>
<td>Mar</td>
<td>1.799</td>
<td>3.749</td>
<td>32.0%</td>
</tr>
<tr>
<td>Apr</td>
<td>3.597</td>
<td>3.597</td>
<td>0.0%</td>
</tr>
<tr>
<td>May</td>
<td>3.683</td>
<td>3.683</td>
<td>0.0%</td>
</tr>
<tr>
<td>Jun</td>
<td>2.298</td>
<td>1.979</td>
<td>16.1%</td>
</tr>
<tr>
<td>Jul</td>
<td>2.375</td>
<td>1.636</td>
<td>32.0%</td>
</tr>
<tr>
<td>Aug</td>
<td>2.375</td>
<td>1.799</td>
<td>32.0%</td>
</tr>
<tr>
<td>Sep*</td>
<td>3.663</td>
<td>1.005</td>
<td>264.5%</td>
</tr>
<tr>
<td>Oct</td>
<td>3.818</td>
<td>4.643</td>
<td>−17.8%</td>
</tr>
<tr>
<td>Nov</td>
<td>4.913</td>
<td>4.693</td>
<td>−20.58%</td>
</tr>
<tr>
<td>Dec</td>
<td>4.361</td>
<td>4.361</td>
<td>0.0%</td>
</tr>
<tr>
<td>Annual</td>
<td>43.61</td>
<td>37.53</td>
<td>16.2%</td>
</tr>
<tr>
<td>Error</td>
<td>$9.45 \times 10^{-4}$</td>
<td>$9.45 \times 10^{-4}$</td>
<td>0.0%</td>
</tr>
<tr>
<td>% Error</td>
<td>19.0%</td>
<td>19.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Figure 5.4: Comparison of annual energy production estimates for 7 turbines at Mt. Raimer, 2001.
between the log and Weibull predictions for Mt. Rainer (16.2%) is smaller than the
difference for Brodie Mountain (23.8%). However, given the uncertainty in the Weibull
prediction (±8.3 million kW-hr), this percentage difference could be anywhere from
38% to−6%. Pinpointing the location of the MTR tower on Truewind’s maps could
reduce this range. Nevertheless, the log law and Truewind predictions seem to be in
better agreement here than in the Brodie Mountain case (see Figure 4.13 in the previous
chapter)—from which we can draw two conclusions: (1) seasonal c and k values may
make for better yearly energy estimates than annual values do, and (2) the more certain
we are of an anemometer tower’s location on Truewind’s map, the better the Weibull
estimate agrees with the log law estimate and the smaller is the former’s uncertainty. To
be certain of these conclusions, however, we would need to compare log law and Weibull
predictions at several other sites with both seasonal and annual parametric values (see
Section 5.1.4 for one such example). In any case, Truewind estimates seem to accurately
predict energy production to within about 10−20%, so in the absence of anemometer
data we should not hesitate to use Truewind’s Weibull parameters.

5.1.4 Taconic Ridge (TCN) & Notch Road (NCH)

Since Lincoln Labs’ Mt. Rainer data set is partially incomplete—96% of September,
70% of July, and 21% of October data sets are missing, averaging 34% over the six
months or 50 + 34/2 = 67% over the entire year—I ran WindData.gs on the data from
two additional sites: Notch Road (site NCH, elev. 530 m) and the Taconic Ridge (site
TCN, 469 m) (see Figure 5.1). I chose these particular sites in the hope that their
data sets would be more complete and because they are the two highest sites after Mt.
Rainer.2 I discovered that NCH was even worse than MTR: 100% of November, 99.9% of
September, and 35% of the October data sets are missing, averaging 44% in all. TCN on
the other hand is significantly more complete—only 4% of the data intervals are missing
(54% annually), with no month missing more than 9%. Thus, plugging the TCN data
into the Excel spreadsheet used for the MTR data, we come up an annual result of 8.05
million kW-hr. The error in this value is slightly smaller than that of the MTR value—
only ±158 kW-hr—because of the smaller number of counts falling into the 14 m/s wind
speed bin (see Section 5.1.1). The TCN log law estimate is a factor of more than 4
times smaller than the MTR value because the TCN site is located at a significantly
lower elevation. This altitudinal difference evidently has a great enough impact on the
wind speed distribution to significantly reduce the predicted energy production, which
explains wind developers’ desire to make turbine towers as tall as possible and to site
them at high elevations.

Determining the location of the TCN site on Truewind’s New England (GIS) wind
map is roughly as difficult as locating the Brodie site: though Bieringer et al. report a
lat-lon coordinate for the site (42.712° N, −73.249° W), we have no other description
of where it is other than a dot on the map in Figure 5.1, which is not located at any
particularly prominent topographical feature. Thus, our search for Weibull parameters

2Excluding Mt. Greylock, which Dr. Bieringer warned did not have a particularly usable data set—
though I have not verified this allegation.
5.2. WIND DIRECTION AT MT. RAIMER

Figure 5.5: Comparison of annual energy production estimates for 7 turbines at Taconic Ridge, 2001 (approximate location: 42.713° N, −73.251° W).

is reduced once again to guesswork, with the most promising values turning out to be \( c = 6.82 \) and \( k = 2.131 \) at 42.713° N, −73.251° W. If we run the Weibull calculations with them we obtain an annual estimate of \( 21.1 \times 10^7 \) kW-hr. The uncertainties in these values are \( \delta c = \pm 1.53 \) and \( \delta k = \pm 3.4 \times 10^{-3} \), which leads to an overall uncertainty in the Weibull energy estimate of \( 1.30 \times 10^7 \) kW-hr (≈ 62%). The two production estimates are plotted in Figure 5.5.

There are two striking features of this plot: the Truewind prediction is a factor of more than 2 times larger than the log law prediction, and the error bars on the Truewind estimate are significantly higher than in the MTR and Brodie cases. These large discrepancies are due to the difficulty of locating the TCN site on Truewind’s map. Clearly, we did not choose quite the right location, since we have greatly overestimated how much energy could be produced. Also, since the terrain in the vicinity of the presumed site is fairly steep, errors in location lead to large errors in elevation, which led to large uncertainties in \( c \) and \( k \). What we can conclude, then, is that if we want to estimate the energy production with Truewind’s maps at a given site, we will only obtain reasonable predictions if we are quite sure that we have the correct \( c \) and \( k \) values from that site.

5.2 Wind Direction at Mt. Raimer

A brief comparison between the measured and Truewind-predicted wind speed distributions at site MTR (Figures 5.6 and 5.7) allows us to evaluate the potential for turbine shading, as well as serving as a double-check on Truewind’s directional accuracy.

Approximately 35% of the wind at Mt. Raimer blows from the NW, with a strong (≈ 13%) component from the SSW. Another smaller component (≈ 8%) blows out of
Figure 5.6: Directional distribution of winds at Mt. Raimer, June–November 2001 (including 4% of the September data that are available). The bars represent the percentage of time the wind blew in each 22.5° sector over the course of the six months. 0° is due north.
5.3. **SUMMARY OF RESULTS**

The Berkshire Mesonet recorded wind data at nine separate locations around Williamstown, MA between June and November 2001. Of these locations, the summit of Mt. Rainer (MTR) is closest to Berlin Pass—the proposed site of the Berlin Wind Project—and thus provides the best estimate of the energy production potential there. The amount of energy that could be produced at Mt. Raimer was predicted in two ways—using the Mesonet wind data and using Truewind’s Weibull distribution parameters. The results of the calculations are presented in Table 5.6.

<table>
<thead>
<tr>
<th>Annual Yield, $10^6$ kW-hr</th>
<th>Log Law</th>
<th>Truewind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error, kW-hr</td>
<td>$\pm 9.45 \times 10^{-4}$</td>
<td>$\pm 8.3$</td>
</tr>
<tr>
<td>% Error</td>
<td>$\pm 0.0%$</td>
<td>$\pm 19.0%$</td>
</tr>
</tbody>
</table>

Table 5.6: Summary of predicted annual energy yields for 7 turbines at Berlin Pass (Berlin, NY, 2001), using the log law and Weibull methods.

The percentage difference between these two estimates is 16 $\pm$ 22$, which verifies that Truewind predictions are fairly accurate. The prevailing NW winds at the site
5.3. SUMMARY OF RESULTS

Figure 5.7: Truewind prediction of directional distribution of winds at Mt. Rainer (42.718° N, −73.286° W). Light bars represent the percentage of time the wind blows in each 22.5° sector over the course of the year, while the dark bars represent the percentage of total energy that would be generated by winds from each sector. Courtesy AWS Truewind LLC.
blow orthogonally to the ridgeline, suggesting that turbine shading may not be a significant problem at Mt. Rainer—and therefore not at Berlin Pass, either. Comparing log law and Truewind estimates at the Taconic Ridge (site TCN) illustrates that the difficulty in locating a site on Truewind’s wind maps can cause considerable over- or under-estimations, and large uncertainties, in annual energy predictions. Nevertheless, we can be confident that using Truewind in the absence of site data will result in reasonable estimates (to within 15–20%), as long as we are sure that the Weibull parameters we obtain are for the actual site in question.
Chapter 6

Black Data Analysis

As part of his senior thesis, Williams College student Thomas Black ’81 collected wind data at Berlin Pass between August 1980 and July 1981. He used these data to estimate monthly energy output for eight separate turbine models on the market at that time. Ultimately, these estimates allowed him to complete an economic study of the proposed wind farm that he hoped would “persuade a group of possibly skeptical trustees” to finance the project ([2], p. 7). Though Black’s data set is hardly complete because of some serious vandalism problems at the site—the best of the three anemometers was only able to record 65% of the year’s data—it is quite valuable because it is from Berlin Pass.

6.1 Black’s Instruments and the In Situ Divider

Black set up a quartet of instruments on and around Berlin Pass: two anemometers and a wind vane were mounted to a 100-foot guyed tower erected at the Pass while a third anemometer was mounted on an existing tower owned by the Motorola Corporation at the summit of Berlin Mountain, about 1.6 km to the south of Black’s tower (Figure 6.1). The Pass tower recorded wind speed and direction at a height of 40 ft and speed at 100 ft—these instruments were nicknamed ‘40PASS’ and ‘PASS100’, respectively. The Berlin anemometer, nicknamed ‘SUMMIT40’, was mounted 40 ft above the ground, as well. A data recorder at the base of each tower collected the data, which Black recovered from the site approximately once every three weeks.

The lexan cup anemometers recorded wind speed by sending a signal to the recorder once per revolution as they closed an electric circuit. The recorder counted up the number of closures over a user-defined time interval, which could then be used to calculate the average wind speed over that interval. The nominal closure rate of the instruments was specified at one closure per mile per hour of wind per second (1 closure/mph·sec), though in practice each anemometer behaved idiosyncratically. Thus, Black sent the instruments to Meteorology Inc. of Altadena, CA for calibration testing. The company measured the number of switch closures per second at various wind speeds for each device and sent the results to Black, who fit 2nd order polynomials to the calibration data ([2], p. 45-6; see Table 6.1). These polynomials allowed him to calculate the wind speeds
6.1. BLACK’S INSTRUMENTS AND THE IN SITU DIVISOR

Figure 6.1: The approximate locations of Black’s met towers. The 40PASS and PASS100 anemometers were located at Berlin Pass proper, while the SUMMIT40 anemometer was situated at the summit of Berlin Mountain, 1.6 km to the south. The translucent irregular polygon is the property owned by Williams College.
(in mph) from the raw number of switch closures per second.

The data recorders only had the capacity to record a total of 9999 switch closures per time interval, which means that if the wind speeds were too great over a given time interval, the recorder’s memory would fill and render that interval’s datum useless. To avoid this problem, Black enlisted the help of Bryce Babcock, Staff Physicist at Williams College, to help him fit an “electronic scaling mechanism” to the instruments that effectively reduced the number of closures the recorder ‘saw’ by a user-specified in-situ divisor. Thus, if the divisor was set to 10 and the anemometer spun 2,500 times over a certain time interval, the data recorder would only record 250 switch closures.

Unfortunately, Black failed to specify in his thesis exactly what divisor he used, which threatened to render his raw data unusable. Beginning to despair one January afternoon, I approached Bryce Babcock, now Coordinator of Science Facilities at Williams, in the hopes that he might remember what value Black had chosen some 23 years before. Though he could not say off the top of his head what divisor had been programmed into the data recorders, he sat down with me and together we pored over Black’s thesis for some hint of where we could discover the itinerant number. Eventually, we found a lead: of the numerous programs Black wrote in Fortran to reduce the raw data, one, called “CALADJ”, was responsible for converting raw data into wind speeds in miles per hour. The key was in this program!

I rushed over to the College Archives in Stetson Hall and asked to see a copy of Black’s thesis. The archivist chuckled and then muttered something about ‘the biggest thesis in the archives’ under her breath. Slightly confused, I waited until, several minutes later, she reappeared and I understood: she carried a massive 300-page tome, complete with Black’s text, the three data sets, all program code, and the complete results of all his calculations. I hurriedly flipped to Appendix F where I found CALADJ, photocopied it, thanked the archivist, and ran back to Babcock’s office where we set about searching for the missing divisor. Struggling with the unfamiliar syntax of Fortran, we eventually hit the jackpot: the program seemed to have divided all data by a factor of 100.

We soon realized, however, that this was not the divisor Black programed into the data recorders—this was a further division by 100 that took place after the data had already been collected. This division, we reasoned, must finish the task of calculating the number of switch closures per second begun by the data recorder. Thus, remembering that the recorder was set to collect data in half-hour time intervals—or 1800-second periods—we determined that Black must have set the unknown in-situ divisor to 18. In this way, the net result of the two divisions—first by 18 in the data recorder and then by 100 in CALADJ—was a division by 1800, giving the average number of switch closures per second over each half-hour period. Using his calibration polynomial (see Equation 6.1), Black could then convert this value to the average wind speed during each interval.

6.2 Energy Production Estimates

The log law and Weibull distribution methods for estimating energy production, which we developed in Chapter 3, can be applied to Black’s data in order to estimate the energy that could be produced by a 7-turbine wind farm at Berlin Pass. Because analyzing all
three of Black’s data sets would be redundant, we will focus our analysis on the PASS100 data.

6.2.1 Log Law

Though Black generally set the data recorders’ time interval to 30 minutes, during one or more periods of vacation he increased the interval to 1 hour so that the recorders’ memory would not overflow while he was gone. For the PASS100 data set he did this only once, between 18 March 1981 at 13:00 and 8 April 1981 at 9:30. Though he makes no mention in his thesis of having changed the in-situ divisor to compensate for the increased time interval during these three weeks, it appears that he ran this data through the same CALADJ program he used for the other half-hour data intervals. We may therefore infer that he changed the divisor to 36 during this period so that, when CALADJ later divided by 100, the result was the number of closures per second over the whole 3600-second time interval. This average value can be assigned to each 30-minute half of the hour-long interval, effectively converting the hourly average back into half-hourly averages.

Table 6.1: Sensor calibration results for Black’s PASS100 anemometer.

<table>
<thead>
<tr>
<th>Switch Closures per second</th>
<th>Test Speed, mph</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>2.6</td>
</tr>
<tr>
<td>4.7</td>
<td>4.9</td>
</tr>
<tr>
<td>10.9</td>
<td>9.9</td>
</tr>
<tr>
<td>22.4</td>
<td>20.1</td>
</tr>
<tr>
<td>27.9</td>
<td>25.1</td>
</tr>
<tr>
<td>39.1</td>
<td>35.1</td>
</tr>
</tbody>
</table>

Instead of running CALADJ, as Black did, to divide the entire data set by 100, we can simply divide the data by 100 in a spreadsheet to convert each half-hour intervals’ datum into an average number of switch closures per second. In order to translate these values into average wind speeds, we fit a 4th order polynomial to the calibration results and obtain the regression relationship in Equation 6.1.

\[ y = 1.261 + .7449x + 4.642 \times 10^{-3}x^2 + 1.216 \times 10^{-5}x^3 - 1.331 \times 10^{-6}x^4, \]  

(6.1)

where \( x \) is the number of switch closures per second, \( y \) is the wind speed in miles per hour, and \( R \approx 1 \).

Now that the raw PASS100 data has been translated into wind speeds, we are finally ready to extrapolate upwards from 100 ft (30.5 m) to 65 m using the log law (Black used the power law, \( \alpha = 1/7 \)), convert from mph to m/s, and calculate energy production (according to Equation 3.3). We should note here that neither Hopkins Memorial Forest (HMF) nor Harriman-West Airport (AQW) temperature data are available for 1980 or 1981, so as a proxy we can simply use 1998 HMF weather data to calculate air densities—and choose turbine power curves—at the Pass. We obtain the results in Table 6.2.
Table 6.2: Monthly and annual log law estimates of energy production for 7 turbines at Berlin Pass (1980-81). The predicted annual total is 173.9% of Williams College’s 2002-2003 energy use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, 10^6 kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan '81</td>
<td>3.609</td>
</tr>
<tr>
<td>Feb '81</td>
<td>4.034</td>
</tr>
<tr>
<td>Mar '81</td>
<td>3.249</td>
</tr>
<tr>
<td>Apr '81</td>
<td>4.577</td>
</tr>
<tr>
<td>May '81</td>
<td>2.863</td>
</tr>
<tr>
<td>Jun '81</td>
<td>2.651</td>
</tr>
<tr>
<td>Jul '81</td>
<td>1.106</td>
</tr>
<tr>
<td>Aug '80</td>
<td>1.259</td>
</tr>
<tr>
<td>Sep '80</td>
<td>2.874</td>
</tr>
<tr>
<td>Oct '80</td>
<td>2.749</td>
</tr>
<tr>
<td>Nov '80</td>
<td>4.857</td>
</tr>
<tr>
<td>Dec '80</td>
<td>3.528</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td><strong>37.36</strong></td>
</tr>
<tr>
<td>% Wms Col</td>
<td>173.9%</td>
</tr>
</tbody>
</table>

Figure 6.2: Monthly log law energy production estimates for Berlin Pass, August 1980–July 1981.
In Figure 6.2 we see roughly the same seasonal variation that we have come to expect: the windiest months occur during the winter time while the calmest months are in the summer. What is interesting here, though, is that April was the windiest month after November. In addition, July is predicted to have the least amount of available energy, whereas at Brodie this honor is reserved for August. The variation in windiest and calmest months is, of course, a manifestation of the larger variation responsible for the $\sim \pm 10\%$ year-to-year differences in annual production estimates such as we observed, for example, in the 1997 and 1998 Brodie data sets.

The uncertainty in the annual production figure can be calculated as described in Chapter 3. Though Black never tells us the magnitude of the error inherent in his wind speed measurements, we shall assume the same $\pm 1\%$ error we have used in previous chapters. Carrying out the calculations, we obtain an error of 0 kW-hr—just as in the Brodie case. This error is vanishingly small because the uncertainty in extrapolated wind speed is not large enough that data points could fall outside their 0.5 m/s-wide speed distribution bins. There are two reasons for this fact. First, the wind measurements were taken from a sufficiently high altitude (30.5 m) that extrapolating the speeds upwards does not introduce significant uncertainty. Second, the $\pm 1\%$ error in wind speeds is not great enough to affect the annual estimate. Thus, a 7 turbines at Berlin Pass have a predicted energy output of 37.4 ± 0 million kW-hr per year, which is 173.9% of Williams College’s 2002-2003 electricity use.

### 6.2.2 Weibull Distribution

Seasonal values of the Weibull $c$ (scale) and $k$ (shape) parameters for Berlin Pass can be obtained from AWS Truewind’s New York wind map. Though Black unfortunately does not divulge the explicit location of his anemometer tower, we shall assume it was situated at the lowest point on the Pass just north of where the ‘jeep road’ crosses the ridgeline (approximately 42.707$^\circ$ W, $-73.296^\circ$ N; see Table 6.3). Since energy depends on wind speed, which in turn depends on hub altitude, using the lowest elevation will assure that any overestimation of the wind resource is as small as possible—and so these $c$ and $k$ values should give us a lower bound on the Truewind-predicted energy. The results of the calculations are presented in Table 6.4 and Figure 6.3.

<table>
<thead>
<tr>
<th>Season</th>
<th>$c$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>9.475</td>
<td>2.229</td>
</tr>
<tr>
<td>Summer</td>
<td>7.514</td>
<td>2.194</td>
</tr>
<tr>
<td>Fall</td>
<td>9.666</td>
<td>2.284</td>
</tr>
<tr>
<td>Winter</td>
<td>11.497</td>
<td>2.486</td>
</tr>
</tbody>
</table>
6.2. ENERGY PRODUCTION ESTIMATES

Table 6.4: Monthly and annual Weibull distribution estimates of energy production for 7 turbines at Berlin Pass (1980-81). The annual total is predicted to be 189.6% of Williams College’s 2002-2003 electricity use.

<table>
<thead>
<tr>
<th>Month</th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>4.700</td>
</tr>
<tr>
<td>Feb</td>
<td>4.246</td>
</tr>
<tr>
<td>Mar</td>
<td>3.488</td>
</tr>
<tr>
<td>Apr</td>
<td>4.344</td>
</tr>
<tr>
<td>May</td>
<td>3.423</td>
</tr>
<tr>
<td>Jun</td>
<td>2.072</td>
</tr>
<tr>
<td>Jul</td>
<td>2.141</td>
</tr>
<tr>
<td>Aug</td>
<td>2.141</td>
</tr>
<tr>
<td>Sep</td>
<td>3.431</td>
</tr>
<tr>
<td>Oct</td>
<td>3.546</td>
</tr>
<tr>
<td>Nov</td>
<td>3.496</td>
</tr>
<tr>
<td>Dec</td>
<td>4.700</td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td>40.73</td>
</tr>
<tr>
<td><strong>% Wms Col</strong></td>
<td>189.6%</td>
</tr>
</tbody>
</table>

Figure 6.3: Monthly Weibull distribution energy production estimates for Berlin Pass (1998) based on AWS Truewind’s New York wind map.
6.2. ENERGY PRODUCTION ESTIMATES

There is nothing particularly remarkable about the monthly distribution of energies presented here: we observe the standard windy winter and calmer summer that we have seen in all other energy production estimates so far. As we should expect, the Berlin Pass prediction \((4.073 \times 10^7 \text{ kW-hr})\), which was made for the lowest point on the property, is less than the Mt. Rainer prediction \((4.361 \times 10^7 \text{ kW-h})\), which was essentially made for the highest point on the property (since the MTR site lies just over the northern property line).

The error in this prediction can be calculated in the standard fashion (see Section 3.2.5). In this case, the difficulty in locating the anemometer site on Truwind’s map (assuming the tower was located at the lowest point on the pass) is not so acute—there are only two reasonable candidate locations. The uncertainty in the parameters, then, turn out to be \(\delta c = \pm 0.0750\) and \(\delta k = \pm 0.0021\). These errors, combined with the usual uncertainty in wind speed of 0.4 m/s, conspire to give an uncertainty of \(\pm 9.110 \times 10^6\) kW-h or \(\pm 22.4\%\) of the annual energy estimate. The magnitude of this error is consistent with those in earlier Weibull predictions. Thus, a 7-turbine wind farm at Berlin Pass could be expected to produce approximately \(40.7 \pm 9.1\) million kW-hr of energy.

6.2.3 Truwind Accuracy

We can now compare the log law and Weibull predictions for Berlin Pass. Taking the log law estimates as the ‘true’ amounts of energy that could be produced at the Pass in 1980–81, we obtain the monthly and annual percentage differences presented in Table 6.5.


<table>
<thead>
<tr>
<th>Month</th>
<th>TW % Diff from Log Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>30.26%</td>
</tr>
<tr>
<td>Feb</td>
<td>5.25%</td>
</tr>
<tr>
<td>Mar</td>
<td>7.37%</td>
</tr>
<tr>
<td>Apr</td>
<td>-26.92%</td>
</tr>
<tr>
<td>May</td>
<td>19.55%</td>
</tr>
<tr>
<td>Jun</td>
<td>-21.85%</td>
</tr>
<tr>
<td>Jul</td>
<td>93.51%</td>
</tr>
<tr>
<td>Aug</td>
<td>69.99%</td>
</tr>
<tr>
<td>Sep</td>
<td>19.39%</td>
</tr>
<tr>
<td>Oct</td>
<td>28.96%</td>
</tr>
<tr>
<td>Nov</td>
<td>-28.02%</td>
</tr>
<tr>
<td>Dec</td>
<td>33.22%</td>
</tr>
<tr>
<td>Annual</td>
<td>9.03%</td>
</tr>
</tbody>
</table>

Since only \(12\%\) of the July anemometer data are missing, the largest difference between the two predictions of \(94\%\) cannot be written off as a fluke caused by anemometer...
malfunctioning, nor is it due to annual $c$ and $k$ values because the New York wind map provides seasonal values. Rather, the reason that Truewind predicts almost twice as much energy as the wind data do is that July 1981 seems to have been much calmer than is normal for the site. A discrepancy of this magnitude cropping up once in a while should be expected; though Truewind does not do such a good job in this particular case, we should expect that natural variation in wind speeds will sometimes result in the availability of energy being either much more or much less than average. Otherwise, the variations in monthly differences range from 70% to -28%, averaging out to an annual difference of only 9%—though because of the uncertainty in the Weibull estimate, this percentage difference could be anywhere from $-15.4\%$ to $33.4\%$, giving a net difference of $9\pm 24.4\%$. The fact that this range of values includes zero renders the two predictions statistically equivalent (though not equivalently precise; see Figure 6.4). In comparison to its analogues from Brodie Mountain (22.7%) and Mt. Raimer (19.0%), this 9% difference supports the finding that Truewind tends to overpredict energy yield by about 10-20%. Thus, any predictions of annual energy production made using Truewind’s Weibull parameters—at least in the northwest Berkshire region—should probably be multiplied by 0.8-0.9 in order to avoid overestimation.

### 6.3 Wind Direction at Berlin Pass

To evaluate the potential for turbine shading, we shall briefly compare the measured and Truewind-predicted directional distributions at Berlin Pass. Black’s 40PASS anemometer was fitted with a wind vane that measured the average wind direction at the Pass during each 30-minute time interval (or 1-hour time interval in the case of ‘vacation
6.3. WIND DIRECTION AT BERLIN PASS

The vane measured direction with a resolution of 16 sectors (22.5° each) numbered 0-15. Though Black does not tell us which sector corresponded to North, we may assume that winds blowing from this direction were assigned to sector 0, as this seems to be standard practice. Instead of recording the directional information in a separate file, however, Black’s data recorder ingeniously combined each wind speed and direction pair into a single datum. To do so, the recorder multiplied the directional sector by 500 and added it to the switch closure number, which had moments before been subjected to an in-situ division by 180,\(^2\) rather than the usual 18, in order that it could not possibly exceed 500. The direction and speed data could then easily be recovered by dividing the resulting datum by 500: the quotient gives the direction, and the remainder the effective number of switch closures. The latter value could then be divided by a further 10 to obtain the average number of switch closures per second. That is,

\[
D = 500d + c/180, \quad \text{(6.2)}
\]

\[
d = \text{Quotient}(D, 500), \quad \text{(6.3)}
\]

and

\[
\bar{c} = c/1800 = \frac{500 \cdot (D\%500)}{10}, \quad \text{(6.4)}
\]

where \(D\) is the resulting datum, \(d\) is the average wind direction (sector 0–15) over the time interval, \(c\) is the number of switch closures over that interval, and \(\bar{c}\) is the average number of switch closures per second (note that, since \(c\) will never exceed 90,000, \(c/180\) can never exceed 500). In this way, we can extract both the directional and speed data from Black’s 40PASS data set. The directional results, plotted in a wind rose created with Microcal’s Origin software, are presented in Figure 6.5.

Not only are almost 44% of the year’s directional data missing, but the resulting wind distribution is strikingly dissimilar to those at Brodie and Mt. Raimer (see Figures 4.15 and 5.6). Whereas at those sites we observed strong NW, fair SW, and weak SE components, Black’s data shows a massive easterly component and not much else. There are three possible explanations for this discrepancy: (1) Our assumption that sector 0 corresponds to north may be wrong. Perhaps instead it is associated with westerly or southerly winds, which would rotate the strong component by 90° or 180° CCW to a NNW-erly or WSW-erly direction. This possibility seems unlikely, however, as setting 0 to be due west would break with convention and because Black gives no motivation for doing so. (2) The second explanation is that I made one or more mistakes in extracting the directional data from the raw 40PASS data set. However, upon double-checking my calculations in the relevant Excel spreadsheet, I was unable to find any mistakes. (3) The third—and most likely—explanation is simply that Black’s directional data are no

\(^1\)Strangely, the ‘vacation data’ in the 40PASS data set is recorded as having begun on 19 March 1981 at 13:00 and having ended at 9:30 on 9 April 1981—one day behind the PASS100 data. There are at least two possible explanations: (1) perhaps the person who transcribed Black’s data from his thesis to the Excel file accidentally shifted the PASS100 data back or the 40PASS data ahead by one day, or (2) the leap year in 1980 caused this shift—though this seems unlikely, as 29 February 1980 is outside the period covered by Black’s data. In either case, the data themselves remained unaffected.

\(^2\)Bryce Babcock and I determined that the 40PASS data used a different divisor by carefully examining the CALADJ code.
Figure 6.5: Directional distribution of winds at Berlin Pass, August 1980–July 1981. The bars represent the percentage of the time the wind blew in each 22.5° sector over the course of the year. 0° is due north. These data are likely inaccurate and should probably be ignored.
good. In fact, we know that there should be a moderate southerly component at Berlin Pass because there is a southerly component at Mt. Raimer, directly to the Pass’ north. Also, we would expect that if the predominant winds to the north (at Mt. Raimer) and south (at Brodie Mountain) of the Pass are from the NW, there should be a strong NW-erly component at the Pass as well. Furthermore, the fact that Black does not mention directional distributions anywhere in his thesis perhaps suggests that since the data turned out to be useless he simply chose to ignore them. In the end, then, we will have to rely on AWS Truewind’s directional distribution prediction instead of actual field data (Figure 6.6). This is not so unfortunate, however, since we know from the Brodie and Raimer data sets that Truewind gets directional distributions more or less right.

![Wind Direction at Berlin Pass](image)

Figure 6.6: AWS Truewind-predicted directional distribution of winds at Berlin Pass (42.707° W, −73.296° N). Light bars represent the percentage of time the wind blew in each 22.5° sector over the course of the year, while the dark bars represent the percentage of total energy that would be generated by winds from each sector. Courtesy AWS Truewind LLC.

The wind rose in Figure 6.6 is quite similar—in fact, identical—to the Truewind wind rose for Mt. Raimer (Figure 5.7). Though in reality there are slight directional differences between them, this similarity illustrates that the spatial resolution of Truewind’s direc-
tional distributions seems to be worse than that of its Weibull coefficients. Despite these considerations, however, the Berlin wind rose is at best specific to a particular latitude and longitude and at worst an average distribution taken over the entire site. Thus, it is probably a fairly accurate representation of the true directional distribution at Berlin Pass. In addition, though approximately equal amounts of wind blow from the SSW (18%) as from the WNW (22%), the strongest winds emanate from the WNW—which is why 22% of the energy comes from the WNW and only 13% from the SSW. Therefore, since the Pass runs approximately N-S, turbine shading should not substantially hinder energy production.

6.4 Summary of Results

Between August 1980 and July 1981, Thomas Black ‘81 measured wind speed and direction at Berlin Pass. Though Black predicted the amount of energy that could have been produced by turbines on the market at that time, we made a more modern estimate for today’s 1.5 MW GE turbines. In addition, we calculated the energy yield using AWS Truewind’s Weibull parameters. The results of the two calculations are presented in Table 6.6.

Table 6.6: Summary of predicted annual energy yields for 7 turbines at Mt. Rainer (Berlin, NY, August 1980–July 1981), using the log law and Weibull methods.

<table>
<thead>
<tr>
<th></th>
<th>Log Law</th>
<th>Truewind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Yield, 10⁶ kW-hr</td>
<td>37.36</td>
<td>40.73</td>
</tr>
<tr>
<td>Error, kW-hr</td>
<td>±0.0</td>
<td>±9.1</td>
</tr>
<tr>
<td>% Error</td>
<td>±0.0%</td>
<td>±22.4%</td>
</tr>
</tbody>
</table>

The percentage difference between these two estimates is 9±24.4%, which verifies once again that Truewind’s predictions are reasonably accurate. When predicting energy yield using only Truewind, though, we should multiply the result by 0.8–0.9 to compensate for the 10–20% overestimate. Black’s wind direction data are probably not representative of the directional distribution at the Pass, but we still obtained a fairly accurate directional distribution from Truewind’s New York wind map. The prevailing component blows from the WNW, while a slightly weaker one emanates from the SSW. Thus, turbine shading is not likely to be an enormous problem at the Pass, especially since the strongest winds blow orthogonally to the ridgeline.
Chapter 7

Energy Yield at Berlin Pass

Over the course of the previous three chapters, we have developed methods for estimating: (1) turbine energy yield and (2) the error in those estimates. In cases where on-site anemometer data exists, the log law proved to be the best technique. On the other hand, in cases where no site data can be obtained the Weibull distribution turned out to be the preferred method, so long as site-specific values of the scale ($c$) and shape ($k$) parameters could be acquired from a wind model. With this information in mind, we applied the pair of methods to the Brodie Mountain, Lincoln Labs, and Berlin Pass wind data in an effort to quantify the accuracy of Weibull predictions with respect to log law estimates of energy yield. We finally determined that the Weibull method tends to overestimate annual energy production by approximately 10–20%. Thus, if we: (1) collect Truewind’s Weibull parameters at the location of each of the seven proposed turbine towers at Berlin Pass, (2) calculate each turbine’s energy production, and (3) rescale the results to take into account Truewind’s 10–20% overestimation, we can predict the energy yield of a wind farm at the Pass as accurately as possible given the current lack of site data.

7.1 Truewind’s Accuracy and the $\gamma$ Factor

Before we can estimate energy production, we must clarify what we mean when we say Truewind ‘overestimates’ energy yield by 10–20% with respect to predictions made with wind data using the log law. That is, does it overestimate by 10%, 20%, or somewhere in between, and what is the typical uncertainty in this ‘overestimation factor’? Ideally, we would answer this question by conducting a thorough survey of numerous wind sites and calculating the ‘overestimation factor’ in each case—and in fact, this is analogous to what AWS Truewind has done in order to come up with the uncertainty figure of .4 m/s associated with their wind maps. It is not obvious, however, how this error in wind speed translates into an error in energy prediction, so we can instead use the results obtained in Chapters 4–6 to determine Truewind’s ‘energy yield accuracy’ directly. These results are presented in Table 7.1.

Estimating energy production at the Taconic Ridge (TCN) using the Weibull distribution involved a good deal of guesswork because it was very hard to tell where on
Table 7.1: Percent that Truewind Weibull predictions overestimate log law predictions at each site (from Chapters 4-6), in addition to their average value and uncertainty.

<table>
<thead>
<tr>
<th>Site</th>
<th>% Diff. btwn log &amp; TW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brodie (UMass)</td>
<td>23.8 ± 28.1%</td>
</tr>
<tr>
<td>Mt. Rainer (MIT LL)</td>
<td>16.2 ± 22.1%</td>
</tr>
<tr>
<td>Berlin Pass (Black)</td>
<td>9.0 ± 24.4%</td>
</tr>
<tr>
<td>Taconic Ridge (MIT LL)</td>
<td>161.8 ± 161.2%</td>
</tr>
<tr>
<td>Average</td>
<td>16.3 ± 14.4%</td>
</tr>
<tr>
<td>‘overestimation factor’ γ</td>
<td>0.860 ± 0.11</td>
</tr>
</tbody>
</table>

Truewind’s New England wind map the site was located. As a result, the predicted energy yield is significantly higher than the figure generated from anemometer data, and so for this prediction there is a large uncertainty. Because the difficulty in locating the measurement tower on the map was so much greater in the TCN case than in the other three cases, we may treat the TCN value in Table 7.1 as an outlier and ignore it.

The other three results—from Brodie Mountain, Mt. Rainer, and Berlin Pass—can be used to generate an average Truewind ‘overestimation factor’ as well as an uncertainty therein. Ideally, of course, we would use more than these three data points to do so, but the paucity of data sets available for this region allows only this level of precision. In addition, it may be the case that Truewind predictions overestimate more for some sites than for others in some systematic way. However, given the small number of data sets any investigation into this systematic variation would represent at best a guess. Therefore, we shall instead assume that Truewind over- or underestimates energy production in a manner that varies randomly with geography. With a larger number of data sets, though, future students could perhaps determine how Truewind’s energy predictions vary with site type (see Chapter 10).

We can calculate Truewind’s average overestimation $\mu$ in the following manner:

$$\mu = \frac{Br + R + Be}{3},$$

where $Br$ is the annual average percentage difference between Truewind and log law predictions at Brodie Mountain (23.8%), $R$ is the difference at Mt. Rainer (16.2%), and $Be$ is the same percentage difference at Berlin Pass (9.0%).

The uncertainty in $\mu$, $\delta \mu$, can be calculated by adding in quadrature the uncertainties in each of the three annual average percentage differences:

$$\delta \mu = \sqrt{\left(\frac{\partial \mu}{\partial Br}\delta Br\right)^2 + \left(\frac{\partial \mu}{\partial R}\delta R\right)^2 + \left(\frac{\partial \mu}{\partial Be}\delta Be\right)^2},$$

where $\delta Br$ is the uncertainty in the Br average (28.1%), $\delta R$ the uncertainty in R (22.1%), and $\delta Be$ the uncertainty Be (24.4%).
7.2. ENERGY YIELD AT BERLIN PASS

Applying these equations, we determine that $\mu = 16.3 \pm 14.4\%$. This percentage can be converted into an ‘overestimation factor’ $\gamma$, which can then be used to appropriately scale down any Truewind prediction of energy yield:

$$\gamma = \frac{1}{1 + \mu}.$$  \hfill (7.3)

Plugging in the average, maximum, and minimum values for $\mu$, we determine that

$$\bar{\gamma} = 0.860,$$  \hfill (7.4)

while $\gamma_{\text{max}} = 0.981$ and $\gamma_{\text{min}} = 0.765$, giving $\delta \gamma = \pm 0.11$.

Armed with $\gamma$, we are finally ready to calculate energy yield at Berlin Pass.

7.2 Energy Yield at Berlin Pass

7.2.1 Monthly and Annual Figures

Locating the seven turbine towers on AWS Truewind’s New England and/or New York online wind maps is a daunting task for anyone not in possession of 20/20 eyesight. The maps can only be zoomed in so far, such that the highest zoom level leaves small-scale topographical details (~tens of meters) to the imagination. Tower site determination is a job better suited for Truewind’s ‘Southern New England’ GIS data layer,\(^2\) which can be zoomed in arbitrarily far and which can be overlain with each tower’s position in ESRI’s ArcView software. Weibull parameters (annual averages) at the location of each turbine are presented in Table 7.2, where turbine 1 is at the south edge of the Berlin Pass property and turbine 7 is at the north edge.\(^3\)

The annual c and k values in Table 7.2 are in good agreement with their seasonal analogues presented in Table 5.3. The summer values ($c \approx 7.5$, $k \approx 2.2$) at the pass are somewhat smaller than the annual averages ($c \approx 9.5$, $k \approx 2.1$) while the winter values ($c \approx 11.5$, $k \approx 2.5$) are somewhat larger. The spring and fall values ($c \approx 9.5$, $k \approx 2.2$) are approximately equal, as well. Thus, the seasonal NY-map values average out to be about the same as their annual NE-map counterparts. Our annual energy predictions, then, should be quite accurate despite the lack of seasonal c and k values—though the individual monthly predictions are probably not worth taking as seriously, as we learned in earlier chapters (and hence they are neither presented nor plotted). The annual prediction for energy yield is presented in Table 7.3.

We can scale down the 12-month total with the ‘overestimation factor’ $\gamma = 0.860$ to obtain a final prediction of energy yield at Berlin Pass: 34.99 million kW-hr, or approximately 162.9\% of Williams College’s electricity usage in 2002-2003. If we assume a net turbine cost (sale price + installation) of $1.24$ million ($8.65$ million for 7 turbines) and an average wholesale electricity price of $38$/MW-hr,\(^4\) a 7-turbine wind farm could

---

\(^1\)Any Truewind prediction for a site in the NW Berkshires, at least.

\(^2\)This map can be obtained from the Massachusetts Technology Collaborative, http://www.mtpc.org/.

\(^3\)I obtained these parameters from Nicholas Hiza ’02, who extracted them from Truewind’s GIS wind map, saving me several hours. For this, I am grateful.

\(^4\)Personal communication with Nicholas Hiza, 10 May 2004.
Table 7.2: Truewind’s annual Weibull $c$ (scale) and $k$ (shape) parameters at the locations of the seven proposed turbines at Berlin Pass (between about $42.70^\circ$ N, $-73.30^\circ$ W and $42.72^\circ$ N, $-73.29^\circ$ W). Turbine 1 is at the south edge of the property, while turbine 7 is at the north edge. Values obtained from AWS Truewind’s ‘Southern New England’ GIS wind map.

<table>
<thead>
<tr>
<th>Turbine</th>
<th>$c$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.34</td>
<td>2.133</td>
</tr>
<tr>
<td>2</td>
<td>9.34</td>
<td>2.133</td>
</tr>
<tr>
<td>3</td>
<td>9.66</td>
<td>2.126</td>
</tr>
<tr>
<td>4</td>
<td>9.66</td>
<td>2.126</td>
</tr>
<tr>
<td>5</td>
<td>9.62</td>
<td>2.115</td>
</tr>
<tr>
<td>6</td>
<td>9.50</td>
<td>2.110</td>
</tr>
<tr>
<td>7</td>
<td>9.74</td>
<td>2.106</td>
</tr>
</tbody>
</table>

Table 7.3: Annual Truewind (Weibull) predictions of energy yield for 7 turbines at Berlin Pass. The predicted annual yield is approximately 162.9% of Williams College’s 2002-2003 energy use (21.48 million kW-hr).

<table>
<thead>
<tr>
<th></th>
<th>Energy, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-Month Total</td>
<td>40.70</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.860</td>
</tr>
<tr>
<td>Energy Yield</td>
<td>34.99</td>
</tr>
<tr>
<td>% WC Energy</td>
<td>162.9%</td>
</tr>
</tbody>
</table>
7.2. ENERGY YIELD AT BERLIN PASS

pay for itself in 6.5 years.

7.2.2 Error Analysis

There are two sources that contribute to the uncertainty in the value of 34.99 million kW-hr: (1) the uncertainty in the overestimation factor $\gamma$ and (2) the same error inherent in the Truewind maps that we have encountered in previous chapters.

To determine the error caused by the uncertainty in $\gamma$, we simply calculate the maximum and minimum values of the energy yield using $\gamma_{\text{max}}$ and $\gamma_{\text{min}}$ and then find their average difference.

$$\delta E_\gamma = \pm \frac{E_{\text{max}} - E_{\text{min}}}{2} = \pm \frac{E \cdot (\gamma_{\text{max}} - \gamma_{\text{min}})}{2} = \pm 4.4 \times 10^6 \text{kW-hr},$$

(7.5)

where $E$ is the 40.70 million kW-hr from Table 7.3.

The error caused by the uncertainty in the Truewind maps can be calculated in the manner described in Chapter 3. There are three facts we should note about this calculation. (1) We will assume that the site of the turbine towers can be located perfectly on the Truewind GIS map, and so there is no error in either $c$ or $k$. This assumption is not particularly bad because we get to define the location of the turbine towers, and so by definition there is no uncertainty in their positions. Also, we can easily overlay a GIS shapefile containing the turbine locations on top of the Truewind map, which makes gathering $c$ and $k$ values from the correct positions quite simple. (2) The errors in $c$ and $k$, which are due to uncertainty in the MesoMap model used to generate the wind maps, conspire to produce an uncertainty in the reported wind speeds. Though we do not know the officially reported error in wind speed for Truewind’s Southern New England GIS map, the $\pm 0.4 \text{ m/s}$ reported for the online New York map makes a reasonable substitute—especially because both maps were generated from same model. (3) Finally, since we would like our results to be as rigorous as possible, we will calculate the error separately for each turbine during each month.

Determining error in this way contrasts with the rougher method used in previous chapters, where we found the error for a single turbine during a ‘representative’ month (e.g. April), multiplied by 7 for the seven turbines, and then multiplied by a further 12 for the twelve months of the year. Because we are using annual $c$ and $k$ values, the individual monthly errors are not particularly meaningful—though the annual value is probably not significantly different than it would be if we had used seasonal parameters. Thus, if we calculate the annual error we obtain $\pm 3.7 \text{ million kW-hr}$.

The errors from uncertainty in both $\gamma$ and Truewind’s maps can, finally, be combined to obtain the net error in the annual energy production figure. The maximum and minimum net error occurs when the two separate errors reinforce each other, so

$$\delta E_{\text{tot}} = \delta E_\gamma + \delta E_{\text{TW}}.$$  

(7.6)

Thus, our prediction of energy yield at Berlin Pass becomes $34.99 \pm 8.08 \text{ million kW-hr}$, or $162.9 \pm 20.5\%$ of Williams’ 2002–2003 electricity usage,\(^5\) and the estimated payback time becomes $6.5 \pm 1.6 \text{ years}$.

\(^5\)It is not clear that these two uncertainties are in fact independent of each other. The error in $\gamma$
7.2.3 Comparison with Black’s Thesis Data Prediction

In this section we briefly compare the log law prediction made from Black’s thesis data to the ‘rigorous’ Truewind prediction made in this chapter. This comparison is outlined in Table 7.4.

Table 7.4: Annual production estimates and errors for 7 turbines at Berlin Pass made with Black’s thesis data (log law) and the Truewind maps in this chapter.

<table>
<thead>
<tr>
<th></th>
<th>Black Data, $10^6$ kW-hr</th>
<th>Truewind Prediction, $10^6$ kW-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annual Yield</strong></td>
<td>37.4</td>
<td>34.99</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td>±0.0</td>
<td>±8.08</td>
</tr>
<tr>
<td><strong>% Error</strong></td>
<td>±0.0%</td>
<td>±37.6%</td>
</tr>
</tbody>
</table>

![Energy Yield Comparison](image)

Figure 7.1: Comparison of energy yield for 7 turbines at Berlin Pass predicted by Black’s thesis data and by Truewind’s maps.

Though the Truewind prediction, once reduced by the ‘overestimation factor’ $\gamma$, is smaller than the estimate made with Black’s thesis data, the error in the former is large enough to render their difference insignificant. This result is of course reassuring because it illustrates, once again, that Truewind estimates are in reasonable agreement with actual wind data. It is also interesting to note that our Truewind prediction underestimates the data-derived prediction. This is the first time we have encountered this behavior; in all other cases, Truewind has overestimated energy yield. Of course, ultimately comes from the uncertainty in the Brodie, Berlin, and Rainer Truewind estimates, and so it actually already takes into account the ±0.4 m/s error in Truewind’s maps. If we ignore the latter source of error, we obtain an uncertainty of only 4.4 million kW-hr. Future research could examine how these two errors are related to each other.
this is not all that surprising since we designed $\gamma$ to keep Truewind predictions from being too large.

### 7.3 Wind Direction at Berlin Pass

Figure 7.2: AWS Truewind-predicted directional distribution of winds at Berlin Pass (approx. 42.707° W, −73.296° N). Light bars represent the percentage of time the wind blew in each 22.5° sector over the course of the year, while the dark bars represent the percentage of total energy that would be generated by winds from each sector. Courtesy AWS Truewind LLC.

Given the fact that the directional information encoded into Black’s 1981 thesis data is either unrecoverable, useless, or both, the only way to predict the directional distribution of winds at Berlin Pass is to use AWS Truewind’s wind rose. For the sake of completeness, then, I reproduce Figure 6.6 here (as Figure 7.2).

In comparing Truewind’s rose diagrams to the measured directional distributions at Brodie Mountain (Chapter 4) and Mt. Rainer (Chapter 5), we noted that although Truewind correctly predicts a fairly strong NW component, it sometimes tends to overestimate the strength of the winds blowing out the SW ($\sim 11\%$ vs. $\sim 6\%$ in the Brodie
7.4 Summary of Results

In this chapter, we applied the results of previous chapters to make a careful and rigorous prediction of the energy yield at Berlin Pass, the site of the Berlin Wind Project. The result of these calculations is presented in Table 7.5.

Table 7.5: Predicted annual energy yield and payback time for a 7-turbine wind farm at Berlin Pass.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Yield at Berlin Pass</td>
<td>34.99 ± 8.08 million kW-hr</td>
</tr>
<tr>
<td>% Williams College energy use</td>
<td>162.9 ± 20.5 %</td>
</tr>
<tr>
<td>Payback time</td>
<td>6.50 ± 1.59 years</td>
</tr>
<tr>
<td>Truewind Accuracy in Berkshires/Taconics</td>
<td>16.3 ± 14.4 %</td>
</tr>
</tbody>
</table>

I believe this figure to represent the best possible estimate of energy yield for a 7-turbine wind farm at Berlin Pass given the lack of a complete anemometer data set from the site. As wind speeds fluctuate from year to year, we should expect the predicted value to vary by as much as an additional ±10%. Furthermore, since the prevailing winds at the Pass blow from the WNW and the ridgeline runs NNE-SSW, turbine shading should not cause substantial energy losses—though there would likely be some losses from a moderate SSW wind component. In addition, I estimate that the wind farm could pay back its $8.65 million cost\textsuperscript{7} in electricity sales within roughly 6.5 ± 1.6 years, assuming an average wholesale electricity price of $38/MW-hr\textsuperscript{8}. To improve on this estimate any further would require a year-long Berlin Pass anemometer study.

\textsuperscript{6}\textsuperscript{6}I leave the calculation of the magnitude of these losses to a future student.

\textsuperscript{7}\textsuperscript{7}The figure excludes the cost of power lines from the wind farm to the grid, the cost of building a road from the old ski area parking lot to Berlin Pass, permitting fees, the cost of conducting the requisite environmental studies, etc.—it represents only the manufacturer’s quoted turbine + installation price.

\textsuperscript{8}\textsuperscript{8}Personal communication with Nicholas Hiza, 10 May 2004.
Chapter 8

A Met Tower on the Roof

In July 2003, the Williams College Department of Geosciences took delivery of a wind measurement tower that it hoped to erect at Berlin Pass as soon as it could obtain the necessary permits from the town of Berlin, NY. In the meantime, the department decided it would be wise to test the equipment so that, once the permits were granted, no time would be wasted in making sure the equipment was functioning properly. This responsibility fell to me, and by late November I had mounted the instruments on a tower atop the roof of the Morley Science Center at Williams College (Figure 8.1). In this chapter, I give a brief account of the testing, mounting, and monitoring of these instruments. In addition, I present preliminary wind data from the roof for the period November 2003–April 2004.

8.1 Equipment List & Initial Testing

The Department of Geosciences purchased its wind measurement equipment from NRG Systems, Inc. of Hinesburg, VT. The delivery included:

- A 50-meter meteorological tower (‘met tower’), in sections\(^1\)
- Four ‘#40 Maximum’ cup anemometers
- Two ‘#200P’ wind direction vanes
- One ‘#110S’ temperature sensor (i.e. thermometer) with radiation shield
- One ‘Symphonie’ data logger
- One AMPS Analog-enabled ‘iPack’\(^2\)
- One photovoltaic (PV) assembly for the iPack

\(^1\)As we were not yet ready to install the equipment at Berlin Pass, the met tower was put into storage at Williams College’s Mt. Hope Farm storage area.

\(^2\)The iPack is an internet communications accessory for the Symphonie data logger that allows for wireless transmission of wind data via AMPS cellular phone service. I did not test this module or its PV/antenna systems.
8.1. EQUIPMENT LIST & INITIAL TESTING

Figure 8.1: The approximate location of the MSL met tower on the campus of Williams College.
• One antenna assembly for the iPack
• A steel shelter box to house the data logger and iPack
• Two 16-MB MMC data cards
• One MMC card reader for use with a PC
• Various accessories (hose clamps, headphones, manuals, batteries, wires, desiccant pack, etc.)

I performed initial testing of the equipment in the lab. In order to determine whether the data logger could produce output intelligible to the PC, I connected various combinations of instruments to the logger and blew air at them with an electric fan. After several unsuccessful runs, I managed to convince the computer to read and display the ‘fan data’ using NRG’s ‘Symphonie Data Retriever’ (SDR) software.

Though I did not explicitly test the iPack, I did learn the following: since it is an AMPS model (as opposed to a GSM or dial-up model), it needs to be assigned its own AMPS analog cellular phone account, which can be obtained from any cellular service provider that still allows analog transmission (e.g. Verizon, AT&T, etc.). In order to set up such an account, one must give the service provider the iPack’s ESN (Electronic Serial Number; 214–00 289628). Once the account has been initiated, the iPack transmits wind data—which arrives as an email attachment—one per day.

8.2 The MSL Roof Met Tower

8.2.1 Installation

Once all the equipment worked to satisfaction, Prof. David Dethier of the Geosciences department retrieved a surplus 10-m met tower from the Hopkins Memorial Forest (HMF) with which we could mount the instruments on the roof of the Morley Science Laboratory (MSL). Since much of the equipment used to attach the instruments to the tower had been stored away with the 50-m tower, Larry Mattison, one of the two college machinist/model-makers, agreed to make a few simple rings and rods that we could use to mount the sensors to the tower. In addition, since the MSL roof is studded with sturdy tether posts, Larry designed and built a ‘mast holder’ that firmly clamps the tower in place so that it can withstand strong winds (see Appendix D for technical diagrams). Data collection began on 25 November 2003, when I erected the tower on the MSL roof with the help of seven brave others\(^4\) (Figure 8.2).

\(^3\)These posts are used by the window washers who clean the skylights of the Schow Science Library, I am told.

\(^4\)The group included three professors, three members of the department of Buildings and Grounds, Larry Mattison, and myself.
8.2. THE MSL ROOF MET TOWER

8.2.2 Preliminary Results

Since installation, the anemometer tower has collected wind speed, wind direction, and temperature data from its position on the northwest corner of the MSL roof. The batteries have remained in ‘good’ condition (∼ 1.5 V) during this time, and all the instruments have functioned perfectly—except for the upper wind vane, which ceased to work at about 19:40 on 24 February 2004. The loss of this sensor’s data does not make our records of wind direction discontinuous, however, since the lower vane continued to function properly over the course of the entire experiment.

Preliminary results, including a speed distribution, hourly average wind speeds, and a wind rose—all created with NRG’s SDR software for the period 25 November 2003–28 April 2004—are presented in Figures 8.3–8.5. As we expect, the mean wind speed in the valley (2–3 m/s; 230 m a.s.l.) is much less than at Berlin Pass (8–9m/s; 670 m a.s.l.). In addition, the predominant winds seem to blow out of the NW/WNW, which is in good agreement with both Truewind’s predictions and the Brodie and Raimer data sets.

Figure 8.2: The author and Larry Mattison performing final checks of the anemometer tower on the roof of the Morley Science Laboratory, Williams College, Williamstown MA, 25 November 2003.
Figure 8.3: Wind speed distribution at the roof of the Morley Science Laboratory, 25 November 2003–28 April 2004.
Figure 8.4: Hourly wind speed averages at the roof of the Morley Science Laboratory, February 2003. Note that speed fluctuates with a period of ~1 day, particularly towards the end of the month.
Figure 8.5: Directional distribution of winds at the roof of the Morley Science Laboratory, 25 November 2003–28 April 2004. Grey bars represent the percentage of time the wind blew from each 22.5° sector, while the black bars represent the percentage of total wind energy emanating from each sector.
Chapter 9

Visual Impact

A visual impact study of the Berlin Wind Project is necessary: (1) as a step in the permitting process and (2) because it allows local residents to carefully weigh the costs and benefits of the project. In this chapter, I describe the methods and tools I employed in evaluating the visual impact of the BWP in sufficient detail that others can verify my results and/or perform a similar study for another proposed farm.

I accomplished the study using a suite of geographical-information-system (GIS) and image-editing software, including ESRI’s ArcMap and ArcScene and Adobe’s Photoshop and ImageReady. I used a hand-held GPS receiver (Garmin’s eTrex) and a digital camera (Olympus’ D-490 Zoom) to gather data that I input into this software. Additionally, I performed several experiments to calibrate the camera. The results—a set of about 60 images that represent possible views of a 7-turbine wind farm at Berlin Pass—are presented in Appendix E.

9.1 Viewshed & Image Collection

The first step in assessing the wind farm’s potential visual impact is to map the locations from which the turbines would be visible. These locations, defined as the set of points from which an observer could see any of the turbines on a clear day, are collectively known as the viewshed. When plotted on a map, the places where the viewshed intersects regions with heavy human traffic or particularly scenic vistas are the places that are most ‘visually sensitive’. It is from these regions that digital images need to be collected so that potential views of the turbines can be created.

In order to calculate the project’s viewshed, Nicholas Hiza ’02 and I used ESRI’s ArcMap software. We specified the locations and heights (65 m) of the seven turbines and ran an algorithm that calculated, for every point on the map, whether there would be a direct line of sight to any of the seven turbines (assuming no vegetation cover1). Points from which no turbines would be visible ArcMap labeled as red, and points from which at least one turbine would be visible it labeled as green (Figure 9.1). We

1In traveling to various places on the map to collect digital images, I discovered that at several of them Berlin Pass was completely obscured from view by vegetation. Thus, the viewshed presented in Figure 9.1 represents an overestimate of the area from which the turbines would be visible.
Figure 9.1: The viewshed of the Berlin Wind Project’s 7 proposed turbines. Regions colored green indicate that at least one turbine would be visible; red areas indicate that no turbines would be visible. The green region represents an overestimate of the turbines’ impact because it was made assuming no vegetation cover. The red circle, which describes an area 20 km in radius, encloses all of the study’s 45 numbered viewpoints (Figure 9.2).
then overlaid the color-coded viewshed on an existing topographical map in order to determine which visually sensitive areas we would designate as image collection points; examples include major roads, places of residence, schools, workplaces, trails, recreation areas, scenic viewpoints, and regions with relatively high population density.

The 45 separate image collection points we identified all lie within a 20 km radius of Berlin Pass, ranging from the hairpin turn on Route 2 in North Adams, MA to the top of Mt. Greylock; from Williamstown, MA to Pownal, VT; from the Taconic Crest Trail—including shots from the wind site itself—to the towns of Petersburgh and Berlin, NY and as far west as Grafton Lakes State Park in Grafton, NY (Figure 9.2; see Appendix E for a complete list of collection points as well as the images generated for those points).

![Image of map showing viewshed and collection points]

Figure 9.2: The 45 points for which views of the proposed wind farm were generated. Light blue points correspond to locations from which the turbines would be visible; beige points correspond to locations from which the turbines would not be visible. Berlin Pass, the site of the wind farm, is outlined by a dashed line in the center of the figure. The translucent irregular polygon is the property owned by Williams College. For the images associated with these points, see Appendix E.

Over the course of the summer of 2003, I drove or hiked to each collection point with the GPS receiver, the digital camera, and a compass and maps—so that I would be sure
9.2. THE DIGITAL CAMERA

The next step in the study is to determine how tall the turbines should be, in pixels, for each image. To do so, we need to know: (1) the distance to the turbines, (2) what angle the turbines subtend from that distance (Figure 9.3), and (3) how many degrees of the field of view each pixel in the digital camera records. The first we can calculate from the waypoints loaded into ArcMap—we measure the horizontal (2D) distance, as well as the difference in elevation, between each point and Berlin Pass, and then combine them using the Pythagorean theorem to obtain the 3D distance $d$.

$$h \quad \square \quad d$$

Figure 9.3: Schematic diagram of the angular size of a turbine.

If we assume the triangle in Figure 9.3 is a right triangle, then $\alpha$ is the inverse tangent of $h/d$. However, since $h/d \ll 1$, $\alpha$ is very small and $\tan(\alpha) \approx \alpha$. In degrees, then, the turbine subtends an angle of

$$\alpha_{\text{deg}} = \frac{180}{\pi} \frac{h}{d}. \quad (9.1)$$

The final piece—the number of degrees each pixel in the camera’s CCD records—can be obtained theoretically or by experiment. I shall describe the theoretical method first and then show that the experimental method produces results that are in reasonable agreement with the theoretical calculations.

9.2.1 Angular Pixel Size: Theoretical

To begin, we briefly review some basic optics. In Figure 9.4, $f$ is the focal length of the camera, $x$ is the size of the image on the camera’s CCD, and $\beta$ is the angular size of

\footnote{OziExplorer: http://www.oziexplorer.com/}
the image. A smaller focal length means the camera is zoomed out, while a larger one means it is zoomed in.

![Diagram of a camera’s lens.](image)

**Figure 9.4:** Diagram of a camera’s lens.

If we first determine the number of degrees each millimeter of the CCD records and then multiply by the number of pixels per mm, we can calculate the number of degrees each pixel records. That is,

\[
L = \frac{\text{degrees}}{\text{mm}} \times \frac{\text{mm}}{\text{pixel}},
\]

(9.2)

where \(L\) is the number of degrees per pixel. As above, \(x/f \ll 1\), so

\[
\beta_{\text{deg}} = \frac{180 x}{\pi f}
\]

(9.3)

and

\[
\frac{\beta}{x} = \frac{180}{\pi f}.
\]

(9.4)

When the Olympus D-490 camera is zoomed out \(f = 5.4\) mm and so there are 10.61°/mm.

In order to extract the value of \(L\), we need to know the size of a pixel in meters. Sony, the maker of the CCD in the D-490, publishes data sheets containing this information.\(^3\) In the D-490, it turns out, each pixel is a 3.275 μm = .003275 mm square. Hence, when the lens is zoomed out, the angular size of a pixel in the D-490 is \(L = .03475^\circ/\text{pixel}\). Similarly, we can calculate that when the lens is zoomed in, \(L = .01158^\circ/\text{pixel}\). The characteristics of the D-490 are summarized in Table 9.1.

**The Lens’ Distortion**

It is important to note that these \(L\) values is not constant over the entire CCD because of distortions introduced by the lens’ curvature. These distortions either slightly magnify or shrink the edges of the image with respect to its center, depending on whether the

camera is zoomed in or out (respectively). It is not clear whether this difference in
distortional behavior at different focal lengths is characteristic of all lenses, but it was
found experimentally to be the case for the Olympus D-490, at least.

As I was concerned that this distortion might make the eventual images inaccurate, I
performed an experiment to assess the magnitude of the problem. I collected two digital
images of a meter stick—one zoomed in and the other zoomed out—and measured the
width of a centimeter, in pixels, at the center and edge of each image. For the zoomed-
out image, I determined that at the center a centimeter measured 17 pixels, while at the
edge it measured 15 pixels, which is a discrepancy of 11.8%. For the zoomed-in image,
at the center a centimeter was 46 pixels wide, while one at the edge was 48 pixels wide,
amounting to a difference of only 4.3%.

Although the distortional effect clearly exists, an 11.8% difference in angular pixel
size is not large enough to invalidate the approximation that angular pixel size is con-
stant over the entire CCD. In addition, in almost every image I collected the wind site
comprises no more than the center 25% of the field of view, which means that the most
pronounced distortional effects are outside the area where precise knowledge of angular
pixel size is required. Thus, we can ignore the lens’ distortional effects on the images
and assume the value of 10.61 °/mm is constant over the entire CCD.

### 9.2.2 Angular Pixel Size: Experimental

To determine the angular size of the D-490’s pixels experimentally, it is necessary to
take a photograph of an object of known size from a known distance away, calculate its
angular extent, and then measure its length in pixels using Photoshop. For simplicity,
I chose a meter stick and took zoomed-in and -out photos from distances of both 5 and
10 meters. The experimental setup, as seen from above, is shown in Figure 9.5.

Here, \( l \) is the length of the meter stick, \( d \) is the distance from the camera to its center
(either 5 or 10 m), and \( \theta \) is the angular size of the stick. In this situation, we cannot
approximate the triangle as a right triangle or use the small angle approximation for \( \theta \)
because here \( d \sim l \). Thus, we have

\[
\frac{l/2}{d} = \tan(\theta/2),
\]

(9.5)
9.3. Creating the Images

Figure 9.5: The experimental setup for determining the angular pixel size of the D-490’s CCD.

so \( \theta = 2 \tan^{-1}(l/2d) \). For \( d = 10 \) m when the camera is zoomed in, the length of the stick as measured in Photoshop is 170 pixels, giving

\[
L = \frac{1m}{170 \text{pix}} \cdot \tan^{-1}\left( \frac{1m}{2 \cdot 10 \text{m}} \right) = 0.03368^\circ/\text{pixel}. \tag{9.6}
\]

This figure is in agreement with the theoretical prediction of \( 0.03475^\circ/\text{pixel} \) to within 3.1%. From \( d = 5 \) m, the stick is measured to be 340 pixels long and we calculate \( L \) to be \( 0.03369^\circ/\text{pixel} \). Here, the measured size is in agreement with the theoretical calculation to within 3.3%. Since, in both cases, the theoretically and experimentally determined values are in such close agreement, we can be confident that the size of the turbines in the resulting images will be quite accurate.

9.3 Creating the Images

Combining the results from above, we can calculate the size of the turbines, in pixels, for each image:

\[
H = \alpha_{\text{deg}} \cdot L = \frac{180}{\pi} \frac{h}{d} L \text{ pixels}, \tag{9.7}
\]

where where \( H \) is the size of the turbines in pixels, \( h \) is their size in meters, \( d \) is the distance from the waypoint to the site, and \( L \) is the angular pixel size. When we later inserted the turbines into the photos, we rounded their size up to the nearest pixel so that, if anything, the images would represent an overestimation of the turbines’ visual impact.

To obtain turbine images to insert into the photos, Hiza created an accurately-proportioned 3D model of a turbine in Corel's ‘Bryce’ modeling software and then exported a set of 2D images of it from various viewpoints.

Next, to determine exactly where the turbines should sit on the ridgeline in each image, I employed ESRI’s ArcScene software, which is a 3D GIS mapping program. ArcScene allowed me to do two things. First, I input the positions of the seven turbines on the ridgeline and told ArcScene that they protruded from the ground by 65 meters. ArcScene then represented the turbines with seven 65-meter tall rectangles sticking off Berlin Pass. I could then use ArcScene to generate exactly the same viewpoint of the
9.4 CONCLUSIONS

Pass depicted in each raw image. I next exported that view, which included the 65-m tall rectangular ‘turbines’, as a 2D image that could be overlain on top the raw image. In this way, the rectangles in the overlay indicated where the turbines should be placed in the image (see Figure 9.6).

I then imported a 2D turbine image into Photoshop, resized it to the calculated height of $L$ pixels and lined it up with the top of the 65-meter rectangles in the ArcScene overlay. In many cases, the bottom of the turbine image lined up nicely with the bottom of the rectangle, although often a portion of the rectangle was eclipsed by some intervening feature of the topography—which means that the base of the turbine tower would be invisible from that particular viewpoint. In these situations, I truncated the 2D turbine image so that its height was the same as that of the eclipsed rectangle, and then lined it up with the eclipsing ridge.

Fearing that perhaps some error was being introduced by this truncation method, I took advantage of ArcScene’s ability to make the landscape transparent, which allows partially obscured objects—including the 65-meter rectangles—to be seen from behind the topographical features that obscure them. To test the turbine truncation technique, I exported two 2D images taken from exactly the same point of observation: one with an opaque landscape, and the other with a transparent landscape. In Photoshop, I carefully lined these two overlays up with each other and with raw image. When I then placed an appropriately sized turbine on top of the transparent layer, I verified that in fact the method is sound: the turbine lined up to within a pixel or two ($\sim 10\%$) with the base of the 65-meter rectangle that was hidden behind the ridge. No longer having to worry, I repeated the process until all turbines visible from that viewpoint had been added to the image.

The final step in the study was to produce an animated GIF of rotating turbines from each viewpoint. To accomplish this, I used Adobe’s ImageReady software, which is a companion application to Photoshop. I imported the completed Photoshop file into ImageReady and then painstakingly turned the different blade layers on and off in ten separate frames to give the illusion of rotational motion. Once this process was finished, I saved the result as an animated GIF, thus completing the analysis of the wind farm’s visual impact for that particular viewpoint. All told, I created some 59 images for 45 viewpoints,\(^4\) which are presented in Appendix E.

\section{9.4 Conclusions}

While the turbines are predicted to be visible from a number of heavily populated and heavily used regions, from many of those regions they may actually appear quite small—so much so that they might go unnoticed by all but the most careful observers. In any case, it is my hope that the results of this study will provide local residents with the information they need to evaluate the visual impact of the proposed wind farm at Berlin Pass.

\(^4\)I also created a number of animated GIFs, which are not presented here.
Figure 9.6: Determining where to place the turbines. (1) A digital image is collected. (2) An overlay is created using 3D GIS mapping software. Note the rectangles that represent the turbines. (3) The turbines are inserted and the overlay is removed.
Chapter 10

Conclusions & Further Research

10.1 Conclusions

The result of the work presented in this thesis is a single number and a collection of images which, though modest in quantity, represent important progress for the Berlin Wind Project. A careful and rigorous prediction of the energy yield of a proposed 7-turbine wind farm at Berlin Pass provides an annual production estimate of $35.0 \pm 8.1$ million kW-hr, or $163 \pm 21\%$ of Williams College’s 2002-2003 electricity use. Because of natural fluctuations in wind speed, this value could vary by as much as an additional $\pm 10\%$ from year to year. Furthermore, since the prevailing winds at the Pass blow from the WNW and the ridgeline runs NNE-SSW, turbine shading should not cause substantial energy losses—though there would likely be some losses from a moderate SSW wind component. Assuming a net turbine cost (sale price + installation) of $1.24$ million ($8.65$ million for 7 turbines) and an average wholesale electricity price of $38$/MW-hr\(^1\), the farm could pay for itself in, very roughly, $6.5 \pm 1.6$ years. I also determine that AWS Truewind’s wind resource maps predict energy yield with an accuracy of approximately $16.3 \pm 14.4\%$ in the northern Berkshire/Taconic region, and that the maps predict directional distributions quite reasonably.

In addition, we can conclude from the visual impact images that the turbines are likely to be visible from quite a few locations throughout the region. However, from a number of these locations the turbines may appear to be quite small and could remain unnoticed by all but the most careful observers. In any case, these images provide the basis for a thorough and widespread public evaluation of the proposed wind farm’s visual impact.

The results of these studies suggest that the Berlin Wind Project is a viable way to generate significant amounts of electricity. Thus, my recommendation to the College is to continue researching the Project while maintaining an open dialog with the local communities so that, together, they may decide whether the BWP represents a worthwhile pursuit for the region as a whole.

\(^1\)Personal communication with Nicholas Hiza, 10 May 2004.
10.2 Future Research

Though I have attempted, in completing this work, to account for as many details and sources of error as possible, there remain a number of ways in which my work could be extended. The following list, though not exhaustive, outlines a few outstanding problems that future students could address.

- The first, and perhaps most obvious, way to refine the prediction of energy yield at Berlin Pass would be to set up a meteorological tower there to record data (wind speed and direction, as well as air temperature). In order to do so, a permit from the town of Berlin, NY would have to be obtained. To date, however, attempts to obtain the necessary permits have been met with reluctance—and so this course of action may not prove particularly fruitful in the near future. Nevertheless, it is not worth giving up altogether, as attitudes have been known to change over time.

If an anemometer is eventually installed, the resulting data could be correlated with concurrent data from, e.g., Albany, NY, to determine the long-term behavior of the winds at Berlin Pass. This correlation could be achieved using the ‘Measure-Correlate-Predict’ method explained in [10].

- Because of time constraints, I was not able to complete a two-dimensional model of the jet stream with $s$ and $\theta$ dependence in Chapter 2. Thus, a future student could continue where I left off and devise a better analytic model of the wind speed profile.

- The Renewable Energy Research Lab (RERL) at UMass Amherst provides wind data from Brodie Mountain recorded at three heights above the ground (10 m, 25 m, 40 m; see Chapter 4). These data could be used to test whether the log and power laws accurately model wind speed profiles in ‘complex’ terrain such as that at Brodie Mountain.

- In Chapter 7, we assumed that energy predictions made with Truewind overestimate energy yield in some average way that varies randomly by site. In reality, it is more likely that Truewind overestimates production systematically depending on site characteristics such as topography, vegetation, weather patterns, temperature, elevation, and the like. It would be interesting to characterize this systematic variation and determine how it affects the prediction of energy yield at the Pass. Possible data sets to include in this study would be: (1) Lincoln Labs’ Berkshire Mesonet data from the sites I did not consider (see Chapter 5), (2) the Brodie Mountain data sets from most of 1996 and 1999, and (3) any other data set that could be obtained for a region represented on one of Truewind’s maps.

- A future student could attempt to characterize the ‘turbine shading’ that would occur at the Pass and calculate how much energy would be lost as a result. The energy loss would probably be mostly due to the SSW wind component, as opposed to the WNW/NW component, since the ridgeline runs from the NNE to the SSW.
By examining temperature data from a number of local weather stations (Albany, NY; North Adams, MA; Hopkins Memorial Forest, Williamstown, MA; etc.), a student could calculate an average local value of \( l \), the atmospheric temperature lapse rate, for use in air density calculations (see Section 3.2.4).

The necessary paperwork could be submitted to the FAA so that they could complete a free aeronautical study of the proposed wind farm (see Section 3.1.1).

In Section 3.2.1, we accounted for ‘missing’ wind data intervals (caused by icing, anemometer malfunctioning, etc.) by assuming that the wind behaved in the same way as it does during ‘normal’ wind conditions. However, if the majority of the missing data is the result of icing events—i.e. storms—the winds may have been much stronger during the ‘missing’ periods. Thus, one could attempt to determine the average difference between ‘normal’ winds and the ‘storm’ winds that occur during icing events, and thereby characterize the uncertainty in the number of counts \( c_i \) used in determining the error in annual energy yield in Chapters 4–6.

In Section 7.2.2, we identified two sources of error contributing to the uncertainty in the final prediction of energy yield at Berlin Pass: (1) the uncertainty in the ‘overestimation factor’ \( \gamma \) and (2) the error inherent in the Truewind maps. To simplify matters, we assumed these sources of error to be independent of each other, though in reality we know they are related. Future research could involve identifying how they are related and what effects that relation would have on the final uncertainty estimate.

I analyzed the visual impact of the proposed wind farm only during the summer. In the fall, winter, and early spring, when there are no leaves on the trees, it is likely that the turbines would be visible from a greater number of locations. Thus, future research could include a careful assessment of the project’s fall/winter visual impact.

In Chapter 7, I assumed an average wholesale electricity price of $38/MW-hr. In reality, this price fluctuates hourly and also shows long-term trends. A future student could thus carry out a more rigorous economic analysis of the Project than I have presented here in order to better estimate the timescale on which it could pay back its initial \( \sim$8.65 million cost. \)
Appendix A

The Betz Limit and Power Curves

Here we derive the power available in the wind, the Betz Limit (the theoretical ceiling on turbine power production), and power curves for both ideal and real turbines.

A.1 Available Wind Power

Consider air of density $\rho$ flowing freely at speed $U$ through a cylinder of area $A$ (Figure A.1). In a time $\Delta t$, the mass that flows through a cylinder of length $U\Delta t$ is

$$m = \rho AU\Delta t. \tag{A.1}$$

The kinetic energy per unit time passing through the cylinder is therefore

$$P = \frac{1}{2} \frac{m}{\Delta t} U^2 = \frac{1}{2} \rho AU^3, \tag{A.2}$$

which is the total power available in the wind.

---

$^1$After [20], p. 31.

$^2$This derivation borrows heavily from [6], p. 41–45.
A.2 Betz Limit

Wind turbines extract kinetic energy from moving air, and so wind passing through a turbine’s rotor slows down. If we assume that the affected air remains separate from air that does not pass through the rotor, we can draw a boundary between the two masses with circular cross section called the ‘stream tube’ (Figure A.2). Because the air inside and outside the stream tube do not mix, the mass flow rate \( m/\Delta t \) inside the tube must be the same everywhere. Thus, as the air—which we assume to be incompressible—slows down as it approaches the rotor, the cross-sectional area of the stream tube must increase to keep \( m/\Delta t \) constant. Furthermore, since no work is done on or by the air until it reaches the turbine’s ‘actuator disc’, its static pressure must gradually increase from \( p_\infty \) to \( p_d^+ \) to absorb this lost kinetic energy.

![Figure A.2: An actuator disc and stream tube.](image)

Even though kinetic energy is extracted from the air flow, a sudden change in speed at the turbine is technologically difficult because of the enormous forces that would be required. Changing pressure in a stepwise fashion, however, is possible—and in fact, all turbines extract energy this way, as Figure A.2 illustrates.

As the air crosses the actuator disc, there is a sudden drop in static pressure from \( p_d^+ \) to \( p_d^- \) about the ambient atmospheric value of \( p_\infty \). Downstream of the turbine, then, in the region called the wake, the air travels at both lowered speed and pressure. Very far downstream the static pressure must return to \( p_\infty \), and so static pressure gradually rises—at the further expense of kinetic energy. Thus, the stream tube continues to expand downstream of the turbine, and the net effect is a loss of kinetic energy.

Since the mass flow rate \( m/\Delta t \) must be constant across all sections of the stream tube, we can write

\[
\rho A_\infty U_\infty = \rho A_d U_d = \rho A_w U_w, \tag{A.3}
\]
where \( \infty \) refers to conditions far upstream, \( d \) to conditions at the actuator disc, and \( w \) to those far downstream.

The actuator disc can be thought of as inducing a percentage change \( a \) in the free stream speed \( U_\infty \). Thus,

\[
U_d = U_\infty (1 - a).
\] (A.4)

Each packet of air (of length \( U \Delta t \)) that flows through the actuator loses a net amount of momentum

\[
\Delta p = m \Delta v = \rho AU \Delta t \cdot \Delta v,
\] (A.5)

so the average force exerted on the air as it approaches, crosses, and leaves the turbine is

\[
F = \frac{\Delta p}{\Delta t} = \rho A_d U_\infty (U_\infty - U_w).
\] (A.6)

This force is caused entirely by the pressure drop across the actuator disc because the stream tube is otherwise surrounded by air at \( p_\infty \), which exerts no net force. Thus,

\[
(p_d^+ - p_d^-) A_d = (U_\infty - U_w) \rho A_d U_\infty (1 - a).
\] (A.7)

We can rewrite the pressure difference \((p_d^+ - p_d^-)\) in terms of \( U_\infty \) and \( U_w \) by applying Bernoulli’s equation separately to the upstream and downstream regions of the stream tube. This equation states that under steady conditions, the total energy in a flow (kinetic, static pressure, and gravitational potential energy) is constant, provided that no work is done on or by the fluid—and so we must apply the equation twice because the total energy is different in each case. Bernoulli’s equation, then, is

\[
\frac{1}{2} \rho U^2 + p + \rho g h = C,
\] (A.8)

where \( C \) is a constant. Upstream, this reduces to

\[
\frac{1}{2} \rho_\infty U^2_\infty + p_\infty + \rho_\infty g h_\infty = \frac{1}{2} \rho_d U^2_d + p_d^+ + \rho_d g h_d.
\] (A.9)

If we assume the flow to be incompressible \((\rho_\infty = \rho_d)\) and horizontal \((h_\infty = h_d)\), then

\[
\frac{1}{2} \rho U^2_\infty + p_\infty = \frac{1}{2} \rho U^2_d + p_d^+.
\] (A.10)

Similarly, for the downstream flow,

\[
\frac{1}{2} \rho U^2_d + p_d^- = \frac{1}{2} \rho U^2_w + p_\infty.
\] (A.11)

If we subtract these two equations, we obtain

\[
(p_d^+ - p_d^-) = \frac{1}{2} \rho (U^2_\infty - U^2_w).
\] (A.12)

Equation A.7 then reduces to

\[
\frac{1}{2} \rho (U^2_\infty - U^2_w) A_d = (U_\infty - U_w) \rho A_d U_\infty (1 - a),
\] (A.13)
so

\[ U_w = (1 - 2a)U_\infty. \]  \hspace{1cm} (A.14)

This result tells us that half the speed loss takes place upstream of the rotor, and half downstream.

The force on the air, from Equation A.7, then becomes

\[ F = (p_d^+ - p_d^-)A_d = 2\rho A_d U_\infty^2 a(1 - a). \]  \hspace{1cm} (A.15)

This force is entirely concentrated at the actuator disc, so the rate of work done by the force is given by \( F \cdot U_d \). Hence, the power extracted from the air, is

\[ P = F \cdot U_d = 2\rho A_d U_\infty^3 a(1 - a)^2. \]  \hspace{1cm} (A.16)

We define the power coefficient to be

\[ C_p = \frac{P}{\frac{1}{2}\rho A_d U_\infty^3}, \]  \hspace{1cm} (A.17)

where the denominator is the total power available in the absence of the actuator disc (Equation A.2). Thus,

\[ C_p = 4a(1 - a)^2. \]  \hspace{1cm} (A.18)

The maximum value of \( C_p \) occurs when

\[ \frac{dC_p}{da} = 4(1 - a)(1 - 3a) = 0, \]  \hspace{1cm} (A.19)

which gives

\[ a = \frac{1}{3} \text{ or } 1. \]  \hspace{1cm} (A.20)

We know that \( a \) cannot be 1, since extracting 100% of the kinetic energy from the air would violate the conservation of momentum. Therefore, \( a = 1/3 \), and the Betz limit is

\[ C_{p_{\text{max}}} = \frac{16}{27} = 0.593. \]  \hspace{1cm} (A.21)

### A.3 Power Curves

The theoretical maximum power curve is given by the Betz-limited power coefficient times the total energy available in the air. Thus,

\[ P_{\text{max}} = \frac{1}{2}\rho AU^3 \cdot C_{p_{\text{max}}}. \]  \hspace{1cm} (A.22)

The power output of a real turbine, however, is limited by its efficiency \( \eta(U) \). Therefore, a real turbine’s power curve is given by

\[ P = \frac{1}{2}\rho AU^3 \cdot C_{p_{\text{max}}} \cdot \eta(U). \]  \hspace{1cm} (A.23)
Appendix B

WindData.gs Program Code and Sample Output

In this appendix I present the code for WindData.gs, which converts MIT Lincoln Labs’ Berkshire Mesonet data from its GrADS format to a tab-delimited text file. GrADS (Grid Analysis and Display System) is an “interactive desktop tool that is used for easy access, manipulation, and visualization of earth science data” developed by researchers at the Center for Ocean-Land-Atmosphere Studies (COLA) and funded by NASA, NSF, and NOAA. The program, along with extensive documentation and a tutorial, is available for free online.¹

The user can output data from any of the nine Mesonet sites for a single month or for each month in the range June–November 2001. In order to run the program, one must place the WindData.gs file in the same folder as GrADS.exe, launch GrADS, and simply type “run WindData.gs”; the resulting text files are labeled and saved in the same GrADS folder. Also included here is sample command-line output when WindData.gs is executed from within GrADS, as well as the first few lines of a sample file created by the program.

B.1 WindData.gs Code

* This GrADS script outputs GrADS data to a tab-delimited text file.
* It is designed for use with MIT Lincoln Labs’ wind and weather
* data from the northwestern Berkshires (The ‘Berkshire Mesonet’).
*
* Samuel Arons ’04
* Williams College
* (with some help from Paul Bieringer, MIT Lincoln Labs)
*
* Senior Physics Thesis
*
* Last update:

* 3 March 2004

say ''
say 'Welcome to WindData'
say 'Export GrADS data into a tab-delimited text file'
say ''
say 'Sam Arons ‘04'
say 'Williams College'
say ''
say ''

say 'Enter the ID of the station you want data from:'
say ''
say 'hwa: Hariman-West Airport, North Adams'
say 'glk: Mt. Greylock'
say 'tcn: Taconic Crest'
say 'wll: Williamstown land fill'
say 'wtr: Williamstown water tower (no useful wind data from this station)'
say 'sch: Mt. Greylock High School'
say 'frm: The farm site west of Greylock'
say 'nch: Notch Road site'
say 'mtr: Mt. Raimer'
prompt ''
pull _id
* the '_' means 'id' is a global variable
say ''

if (_id = 'hwa')
    _name = 'Harriman-West Airport'
    _lat = 42.69655000
    _lon = -73.1625333
    _alt = 199
    _height = 10
endif
if (_id = 'glk')
    _name = 'Mt. Greylock Summit'
    _lat = 42.63730000
    _lon = -73.1682333
    _alt = 1064
    _height = 20
endif
if (_id = 'tcn')
    _name = 'Taconic Ridge'
    _lat = 42.71178333
    _lon = -73.2489000


_alt = 469
_height = 10
endif
if (_id = 'wll')
    _name = 'Williamstown Landfill'
    _lat = 42.73161667
    _lon = -73.2094333
    _alt = 186
    _height = 10
endif
if (_id = 'wtr')
    _name = 'Williamstown Water Tower'
    _lat = 42.71761667
    _lon = -73.1766333
    _alt = 240
    _height = 20
endif
if (_id = 'sch')
    _name = 'Mt. Greylock High School'
    _lat = 42.67425000
    _lon = -73.2351500
    _alt = 287
    _height = 10
endif
if (_id = 'frm')
    _name = 'Farm Site'
    _lat = 42.67330000
    _lon = -73.1964000
    _alt = 323
    _height = 10
endif
if (_id = 'nch')
    _name = 'Notch Road'
    _lat = 42.67050000
    _lon = -73.1562500
    _alt = 530
    _height = 23.7
endif
if (_id = 'mtr')
    _name = 'Mt. Raimer Summit'
    _lat = 42.71613333
    _lon = -73.2831000
    _alt = 781
    _height = 16.1
endif
say 'Do you want an individual month's worth of data, or the'
say 'whole data set in individual monthly files?'
say 'Enter 1 for a single month, or 2 for the entire data set'
prompt '
pull choice

if (choice = 1)
    say 'Enter month of _id data you want to export'
say '(june - november; use lowercase)'
prompt '
pull mon
say ''

writeFile(mon)
else
    writeFile('june')
    writeFile('july')
    writeFile('august')
    writeFile('september')
    writeFile('october')
    writeFile('november')
endif

function writeFile(month)
    * Open the desired GrADS file
    'open LLdata/’month’.ctl’
    'q file’
    line = sublin(result, 5)
    tmax = subwrd(line, 3)
    time = 1

    * Set file variables for data recording
    'set x 1'
    'set y 1'
    'set t ’time
    'set gxout stat'
* Open file, write header

```hs
res = write(month'_'_id'.txt', 'Lincoln Labs Wind Data')
res = write(month'_'_id'.txt', month', 2001', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', 'Station ID: ' _id, append)
res = write(month'_'_id'.txt', 'Lat: _lat, append)
res = write(month'_'_id'.txt', 'Lon: _lon, append)
res = write(month'_'_id'.txt', 'Elevation: _alt m asl', append)
res = write(month'_'_id'.txt', 'Instrument height: approx _height m above ground', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', 'Time step of data: 5 min', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', 'Original data sample rate: 0.2 Hz', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', 'Reformatting by Sam Arons '04, Williams College', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', 'March 2004', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', ', append)
res = write(month'_'_id'.txt', ', append)
```

* Step through time intervals and record date/time and wind speed/direction

```hs
say ' Recording _id' wind data for 'month'...
```

```hs
while (time <= tmax)
```

* Tests if the time the data point was recorded

```hs
was 00:00 or not, (because the time/date format is
* different in this case). Records the time in
* the variable 'moment', and the date in 'date'.
'q time'
```

```hs
line = sublin(result, 1)
line = subwrd(line, 3)
test = substr(line, 3, 1)

if (test = 'Z')
moment = '00:00'
date = substr(line, 4, 9)
else
moment = substr(line, 1, 5)
```


B.2. SAMPLE COMMAND-LINE OUTPUT

Grid Analysis and Display System (GrADS) Version 1.8SL11
Copyright (c) 1988-2001 by Brian Doty
Center for Ocean-Land-Atmosphere Studies
Institute for Global Environment and Society
All Rights Reserved

Config: v1.8SL11 32-bit little-endian readline sdf/xdf netcdf DODS-enabled lats
printim
B.2. SAMPLE COMMAND-LINE OUTPUT

Issue 'q config' command for more information.

Landscape mode? (no for portrait): y
GX Package Initialization: Size = 11 8.5
run WindData.gs
ga-> run WindData.gs

Welcome to WindData
Export GrADS data into a tab-delimited text file

Sam Arons '04
Williams College

Enter the ID of the station you want data from:

hwa: Hariman-West Airport, North Adams
glk: Mt. Greylock
tcn: Taconic Crest
wll: Williamstown land fill
wtr: Williamstown water tower (no useful wind data from this station)
sch: Mt. Greylock High School
frm: The farm site west of Greylock
nch: Notch Road site
mtr: Mt. Raimer
tcn

Do you want an individual month’s worth of data, or the whole data set in individual monthly files?
Enter 1 for a single month, or 2 for the entire data set
2

Recording tcn wind data for june...
  june finished.

Recording tcn wind data for july...
  july finished.

Recording tcn wind data for august...
  august finished.

Recording tcn wind data for september...
  september finished.
Recording tcn wind data for october...

october finished.

Recording tcn wind data for november...
november finished.

gap-

B.3 Sample Output File

The first few lines of a sample file (june_mtr.txt) created by WindData.gs (‘-9999’ indicates a bad data interval):

Lincoln Labs Wind Data
june, 2001

Site name: Mt. Raimer Summit
Station ID: mtr
Lat: 42.71613333
Lon: -73.2831
Elevation: 781 m asl
Instrument height: approx 16.1 m above ground

Time step of data: 5 min
Original data sample rate: 0.2 Hz
(1 Hz for station hwa)

Reformatting by Sam Arons ‘04, Williams College
March 2004

date time wind spd, m/s wind dir, deg
01JUN2001 00:00 -9999 -9999
01JUN2001 00:05 6.095 203
01JUN2001 00:10 6.351 202.6
01JUN2001 00:15 5.53 199.8
01JUN2001 00:20 4.955 198.1
01JUN2001 00:25 3.75 197.6
Appendix C

The Global Wind Resource

Given the amount of solar radiation incident upon the earth, how much useful energy could theoretically be extracted from the wind? A complete and accurate answer would require that quite a number of ‘messy’ factors be accounted for, such as global topography, ocean currents, and atmospheric heat-capacity gradients, to name a few. The present ‘back of the envelope’ calculation, then, necessarily approximates these factors in a more mathematically tractable manner: the earth is assumed to be a sphere, all solar radiation is assumed to be absorbed by the ground (which is everywhere the same temperature), and the exchange of heat between the earth and its atmosphere is modeled as an ideal heat engine.

C.1 Solar Radiation & Terrestrial Absorption

Since it is the sun’s energy that is ultimately responsible for driving terrestrial winds, we first must determine how much solar energy is incident upon the earth. The luminosity of the sun is \( L = 3.826 \times 10^{33} \text{erg/sec} \), so at the earth’s distance from the sun of 1 AU = 1.496 \times 10^{13} \text{cm} \), the solar radiation’s power per unit area is

\[
F = \frac{L}{4\pi r^2} = 1.360 \times 10^6 \text{erg/sec} \cdot \text{cm}^2 = 1.360 \times 10^3 \text{J/sec} \cdot \text{m}^2. \tag{C.1}
\]

This value is known as the solar constant ([8], p. 67). Since the earth’s radius is 6.4 \times 10^6 \text{m} \), its surface area is \( A = 4\pi r^2 = 5.15 \times 10^{14} \text{m}^2 \). However, only half the earth is exposed to sunlight at any one time. So the total power incident upon the earth is

\[
P_{\text{abs}} = F \cdot \frac{A}{2} = 3.5 \times 10^{17} \text{J/sec}. \tag{C.2}
\]

I assume that all 3.5 \times 10^{17} \text{Watts} are absorbed by the earth’s surface.

C.2 A Giant Heat Engine in the Sky

The mean value of the earth’s surface temperature is around 280 K (9 °C).\(^1\) The temperature of the jet stream, which is at an altitude of about 10 km, is around 220 K

\(^1\)The American Association of Amateur Astronomers; http://www.corvus.com/planets/earth.htm
(-53°C)^2. We can think about the surface of the earth and the jet stream as respectively being the hot and cold reservoirs of a giant ideal heat engine that operates at the Carnot efficiency

$$e_{carnot} \leq 1 - \frac{T_C}{T_H}. \quad (C.3)$$

We can thus calculate the total power that could be extracted while the surface of the earth and the atmosphere attempt to come into thermal equilibrium:

$$P_{extract} = e_{carnot} \cdot P_{abs} = (1 - \frac{T_C}{T_H}) \cdot P_{abs} = 7.5 \times 10^{16} \text{J/sec}. \quad (C.4)$$

## C.3 Extracting Aeolian Energy

Since even a perfect turbine can only achieve the Betz limit of 59.3% efficiency (see Appendix A), the maximum power that turbines could extract from the wind is

$$P_{turb} = e_{Betz} \cdot P_{extract} = .5926 \cdot P_{extract} = 4.4 \times 10^{16} \text{J/sec}. \quad (C.5)$$

Now, if each turbine is rated at 1.5 MW = 1.5 × 10^6 J/sec, we would need

$$n_{turb} = \frac{4.4 \times 10^{16} \text{J/sec}}{1.5 \times 10^6 \text{J/sec}} = 2.96 \times 10^{10} \text{ turbines} \quad (C.6)$$

to extract all of the available wind energy. That is almost 30 billion turbines! At a cost of about $1 million each, the price tag comes out to $30 quadrillion. If each turbine requires an area around it approximately two rotor-diameters (~150 m) in radius, then the total area covered by the turbines would need to be

$$A_{turb} = n_{turb} \cdot \pi r_{turb}^2 = 2.96 \times 10^{10} \cdot \pi \cdot (150 \text{ m})^2 = 2.1 \times 10^{15} \text{m}^2. \quad (C.7)$$

Since the earth’s surface area is only 5.15 × 10^{14} m^2, we would need to cover the surface of approximately four earths with 1.5 MW turbines in order to extract all the solar energy available in wind.

## C.4 United States Energy Consumption

Even if we could extract all this power, how would it compare to the United States’ annual energy consumption? According to the USGS Energy Resources Program, in 1998 the United States consumed a staggering 94.27 quadrillion BTU\(^3\). This converts to

$$94.27 \times 10^{15} \text{BTU} \cdot \frac{1055 \text{ J}}{\text{BTU}} = 9.95 \times 10^{19} \text{J}. \quad (C.8)$$

---

\(^2\)NASA Headquarters; [http://www.hq.nasa.gov/1wgsdi/Atmosphere.html](http://www.hq.nasa.gov/1wgsdi/Atmosphere.html)

If the 30 billion turbines produce energy at their collective maximal rate of \(4.4 \times 10^{16}\) Watts, then in one year they would extract
\[
4.4 \times 10^{16} \text{J/sec} \cdot \pi \times 10^7 \text{sec} \approx 1.4 \times 10^{24} \text{J}, \tag{C.9}
\]
or about 14 thousand times the energy consumed by the United States annually. So even though it would be impractical to try to extract all the energy in the wind, extracting enough to power the entire US would only require an area of
\[
\frac{2.1 \times 10^{15} \text{m}^2}{14,000} = 1.5 \times 10^{11} \text{m}^2 \tag{C.10}
\]
covered by
\[
\frac{2.96 \times 10^{10} \text{turbines}}{14,000} = 2.13 \times 10^6 \text{ turbines}. \tag{C.11}
\]
at a cost of \$2.13 trillion. Since the US’ total land area is\(^4\)
\[
3,537,438 \text{ mile}^2 \cdot \frac{2.59 \times 10^6 \text{m}^2}{\text{mile}^2} = 9.2 \times 10^{12} \text{m}^2, \tag{C.12}
\]
only about
\[
\frac{1.5 \times 10^{11} \text{m}^2}{9.2 \times 10^{12} \text{m}^2} = 1.6\% \tag{C.13}
\]
of the country would need to be covered in 1.5 MW turbines. That is equivalent to an area about half the size of New Mexico.\(^5\)

### C.5 World Energy Consumption

How does the power available in the wind measure up to world energy needs? According to the US Department of Energy, in 2001 the world consumed 404 quadrillion BTU (note that the US consumes almost 25% of the world’s energy)\(^6\). This figure works out to a rate of roughly \(1.3 \times 10^{13} \text{J/sec} = 13.6 \times 10^4 \text{ gigawatts, or about}
\[
\frac{1.3 \times 10^{13} \text{watts}}{4.4 \times 10^{16} \text{watts}} = 0.03\% \tag{C.14}
\]
of the total available wind resource. To extract this energy would require about 9 million 1.5 MW turbines covering an area of \(6.4 \times 10^{11} \text{m}^2\) and costing about \$9 trillion. This area seems large until we realize that it is only
\[
\frac{6.4 \times 10^{11} \text{m}^2}{5.15 \times 10^{14} \text{m}^2} = 0.1\% \tag{C.15}
\]

---


\(^5\) New Mexico’s area is 121,356 mile\(^2\) = \(3.1 \times 10^{14}\text{m}^2\). About half of it would provide the needed \(1.5 \times 10^{11}\text{m}^2\).

of the world’s surface area, if we include the possibility of offshore wind farms. If the
turbines were spaced evenly over the entire globe, their density works out to be

\[
\frac{9 \times 10^6 \text{ turbines}}{6.4 \times 10^9 \text{ km}^2} = 1.42 \times 10^{-5} \text{ turbines/km}^2
\] (C.16)

or 71 thousand km\(^2\) per turbine. It is more likely, however, that the turbines would
be clustered into wind farms within areas of greater wind resource, so the density of
turbines would probably be even lower in most places.

To consider it differently, the current global population is about 6.3 billion people.\(^7\) Thus, per capita there would be

\[
\frac{9 \times 10^6 \text{ turbines}}{3.6 \times 10^9 \text{ people}} = .0014 \text{ turbines/person},
\] (C.17)

or one turbine for every 700 people.

Alternatively, we could perform the calculation on a per-country basis. There are
currently 192 countries recognized by the US Department of State.\(^8\) If each country had
to share the burden of constructing turbines equally, then approximately

\[
\frac{9 \times 10^6 \text{ turbines}}{192 \text{ countries}} = 47,100 \text{ turbines/country}
\] (C.18)

would need to be built. Of course, it is more likely that turbine construction duties
would be proportional to land area or GDP, so larger countries like the United States
would have to build significantly more turbines than smaller countries.

### C.6 Conclusions

Although these calculations are rough, they provide a useful order-of-magnitude estimate
of the global wind resource. The total power that could be extracted from the wind,
under ideal conditions, is about 44 million gigawatts—but to do so an area four times
the size of the earth would have to be covered in 1.5 MW turbines. Clearly this is
impractical. Thankfully, we do not need nearly that much power—in 1998 the United
States consumed energy at a rate of 3,000 gigawatts, while in 2001 the world used 13,600
gigawatts. In order to satisfy the US’ energy energy needs, an area roughly half the size
of the state of New Mexico would have to be covered in turbines, costing about $2.13
trillion. To meet world energy demands, we would need to cover 0.1% of the earth’s
surface in turbines at a cost of about $9 trillion. This figure works out to about one
turbine for every 700 people, or 47,000 turbines per country.

These results show, I hope, that wind power—to say nothing of its non-polluting
and renewable nature—is a worthwhile pursuit. Obviously no one would argue that we

---

\(^7\)US Census Bureau, ‘Total Midyear Population for the World: 1950–2050’;
http://www.census.gov/ipc/www/worldpop.html

\(^8\)US Department of State, ‘Independent States in the World’;
http://www.state.gov/s/inr/rls/4250.htm
should meet 100% of our energy needs with a non-dispatchable source such as wind or solar power. But producing even 5–10% of the energy we need by wind could do us a lot of good in the long run.