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Title: Visualizing Strain and the Rf- $\Phi$  Method with an Interactive Computer Program

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**Abstract:** The Rf- $\Phi$  method is a powerful graphical approach for estimating finite strain of deformed elliptical objects, but one that students commonly find difficult to understand. We developed a program that allows users to explore visually how deforming a set of elliptical objects appears on Rf- $\Phi$  plots. A user creates or loads the ellipses and then deforms them by simple shear, pure shear, or rigid rotation. As the ratio of the long to short axis of the ellipses (Rf) and long-axis orientations ( $\Phi$ ) change in one window, the Rf- $\Phi$  plot continuously and instantaneously updates in another. Users can save snapshots of the deformed elliptical objects and the Rf- $\Phi$  plots to record graphical experiments. The program provides both Rf vs.  $\Phi$  and polar  $\ln(Rf)$  vs.  $2(\Phi)$  plots. The user can 'undeform' ellipses quickly and easily, making it possible to inspect the 'original' shapes and orientations of objects, and to evaluate the plausibility of the determined strain values. Users can export information about the pebbles' shape and orientation to spreadsheets for rigorous statistical analysis. This program is written in Java and so can run on virtually any operating system. Both the source code and the application will be freely available for academic purposes.

September 1, 2009

Dear Bill:

Thanks very much for your great editorial suggestions. I have incorporated all of them with a few minor adjustments.

Sincerely,

Paul Karabinos

1       **Visualizing Strain and the  $R_f$ - $\Phi$  Method with an Interactive Computer Program**

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20

## 21 **Abstract**

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23 deformed elliptical objects, but one that students commonly find difficult to understand.  
24 We developed a program that allows users to explore visually how deforming a set of  
25 elliptical objects appears on  $R_f$ - $\Phi$  plots. A user creates or loads the ellipses and then  
26 deforms them by simple shear, pure shear, or rigid rotation. As the ratio of the long to  
27 short axis of the ellipses ( $R_f$ ) and long-axis orientations ( $\Phi$ ) change in one window, the  
28  $R_f$ - $\Phi$  plot *continuously and instantaneously* updates in another. Users can save snapshots  
29 of the deformed elliptical objects and the  $R_f$ - $\Phi$  plots to record graphical experiments. The  
30 program provides both  $R_f$  vs.  $\Phi$  and polar  $\ln(R_f)$  vs.  $2(\Phi)$  plots. The user can ‘undeform’  
31 ellipses quickly and easily, making it possible to inspect the ‘original’ shapes and  
32 orientations of objects, and to evaluate the plausibility of the determined strain values.  
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34 rigorous statistical analysis. This program is written in Java and so can run on virtually  
35 any operating system. Both the source code and the application will be freely available  
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37

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39

## 40 **1. Introduction**

41           Deformed ellipsoidal objects, such as pebbles and oolites, are common in rocks,  
42 and they offer an intuitive, visually appealing approach for teaching fundamental strain  
43 concepts in structural geology. Students can easily grasp the effect of strain on an initially  
44 spherical object and, with practice, can visualize the fate of initially ellipsoidal markers.  
45 Furthermore, the study of deformed pebble conglomerate and oolitic limestone provides  
46 an excellent opportunity for students to gain experience in data acquisition and error  
47 analysis. They must also confront a host of important problems that plague all attempts to  
48 quantify strain, such as ductility contrast between marker object and matrix, initial shape  
49 and distribution of marker objects, area or volume change during deformation, and the

50 relationship between two-dimensional strain measured in planar sections with the three-  
51 dimensional strain experienced by rocks.

52 Structural geologists commonly exploit elliptical objects in their research to  
53 quantify strain in naturally deformed rocks, understand the development of deformation  
54 fabrics, and examine strain gradients in folds and fault zones. The importance of this  
55 approach to strain measurement has inspired numerous studies to overcome its inherent  
56 limitations, or at least to understand them thoroughly. Through numerical experiments,  
57 Lisle (1979) tested several methods for averaging shape and orientation data to determine  
58 the most accurate for estimating the strain ellipse, and concluded that the harmonic mean  
59 was the most reliable. Hossack (1968) and Treagus and Treagus (2002) also discussed in  
60 detail the problems of determining strain from pebble shapes in a conglomerate.

61 Ramsay (1967) derived the equations of the  $R_f\text{-}\Phi$  method for quantifying finite  
62 strain, and Dunnet (1969) showed how the  $R_f\text{-}\Phi$  method can be used as a practical tool  
63 for strain determination from elliptical objects. As discussed in detail below, the  $R_f\text{-}\Phi$   
64 method assumes an initially random distribution of ellipse long-axis orientations, and a  
65 range of initial long to short axial ratios,  $R_i$  (Table 1). Ramsay and Huber (1983)  
66 presented an especially useful, and well-illustrated, discussion of the  $R_f\text{-}\Phi$  method  
67 making the technique more accessible to researchers and students. Lisle (1985) offered a  
68 very complete and useful treatment of the method. Our contribution is to provide a  
69 program that links deformation of elliptical objects with  $R_f\text{-}\Phi$  plots, giving students a  
70 *visual* explanation of how the method works, and offering students and researchers a tool  
71 to quickly estimate strain from outcrops and samples.

72 Our program provides both the familiar Cartesian  $R_f\text{-}\Phi$  plot and the innovative  
73 polar plot of Elliott (1970) for comparison with the ellipse population. Elliott's (1970)  
74 approach employed a novel "shape factor grid" and a polar plot of  $\ln(R_f)$  vs.  $2(\Phi)$  that  
75 should, in theory, allow assessment of the initial distribution of long axes of elliptical  
76 objects, and estimate the strain. The greatest limitation of this approach is the apparent  
77 complexity of the distribution of undeformed elliptical objects (Boulter, 1976; Paterson  
78 and Yu, 1994). Yamaji (2005) developed an inverse method to overcome some of these  
79 limitations for the special case of a bivariate normal distribution of sedimentary particles.  
80 Another limitation to Elliott's (1970) approach, and a possible explanation for why the

81 method has been underutilized, is the significant difficulty most users have with  
82 visualizing the effect of strain on elliptical objects in polar plots. Our program helps  
83 overcome the latter limitation by showing how Cartesian and polar  $R_f$ - $\Phi$  plots change  
84 during deformation.

85 Lisle's (1985) approach to testing the assumption of an initially random  
86 distribution of pebble long axes was to create a set of "marker deformation grids" using  
87 Cartesian  $R_f$ - $\Phi$  plots, and to examine the distribution pattern of deformed ellipses. Sets of  
88 pebbles that were consistent with the assumption should show a symmetrical pattern  
89 about both the harmonic mean of  $R_f$  and the vector mean of  $\Phi$ . DePaor (1988) developed  
90 another novel and useful approach to the  $R_f$ - $\Phi$  method that uses a hyperbolic net and  
91 symmetry principles to estimate strain from ellipsoids, but our program does not include  
92 hyperbolic plots.

93 Several commercially available drafting programs allow users to create a set of  
94 elliptical objects and to simulate deformation with tools that linearly transform the  
95 ellipses by pure shear, simple shear, and rigid rotation. These programs are very useful  
96 for teaching purposes. Also, several commercial programs are available that permit  
97 researcher to determine strain with the  $R_f$ - $\Phi$  method using axial ratio and orientation data.  
98 Some of these programs incorporate statistical methods to assess the validity of the  
99 assumption of initial random distribution of long axis orientations. We have not  
100 duplicated the capabilities of these programs. Instead, we developed a relatively simple  
101 program that focuses on *visualizing* the relationship between strained elliptical objects  
102 and plots of axial ratio vs. orientation. Our program is complimentary to the existing  
103 software because it is straightforward to examine deformed objects with our program and  
104 then, to export information about the ellipses to data files for use in these programs.

105 Our goal was to create a simple and easy-to-learn interactive computer program  
106 that allows the user to simulate deformation of elliptical objects by pure shear, simple  
107 shear, and rigid rotation. Throughout the linear transformations, Cartesian or polar  $R_f$ - $\Phi$   
108 plots are continuously and instantaneously updated. The advantage of this program is that  
109 the color coding and tracking options make it possible to visualize the distribution *and*  
110 *paths* of points representing elliptical objects on  $R_f$ - $\Phi$  plots. This is especially valuable  
111 for the polar plots of  $\ln(R_f)$  vs.  $2(\Phi)$ , and it helps highlight the potential of this neglected

112 approach. We also show here how the program is used to introduce students to the  $R_f$ - $\Phi$   
113 method, determine strain in natural samples, and simulate ‘retro-deformation’ of samples  
114 to recover the original shapes and orientation of the pebbles for critical evaluation of the  
115 method. This program is written in Java, and so can run on virtually any operating  
116 system. Both the source code and the finished application will be freely available for  
117 academic purposes.

118

## 119 **2. Summary of the Program**

120 The program contains a large display window on the left and a display control  
121 window on the right (Fig. 1A). Ellipses are created in the display area by dragging with  
122 the mouse in editing mode. The user can import a 700 by 700 pixel photograph or other  
123 image as a background, and trace elliptical objects from it (Fig. 1B). Alternatively, text  
124 files containing information about the position, shape, and orientation of elliptical objects  
125 can be loaded. An Excel workbook that serves as a template for creating such files is  
126 included with the program. Several buttons control the appearance of the ellipses and the  
127 scale and position of the display (Table 2). All pebbles in the display area are plotted in  
128 the small  $R_f$  vs.  $\Phi$  plot in the display control window as they are created (Fig. 1A).

129 The user can choose deformation by simple shear, pure shear, or rigid rotation  
130 with the radio buttons (Fig. 1). Once a radio button is chosen, deformation can be  
131 precisely specified or accomplished by click-and-drag with the mouse in the display area.  
132 During simulated deformation, the  $R_f$  vs.  $\Phi$  plot is continuously and instantaneously  
133 updated as ellipses change shape and orientation in the display area. A larger and more  
134 versatile  $R_f$  vs.  $\Phi$  plot appears when the user clicks the Big Cartesian Plot button. The  
135 Big Polar Plot button summons a polar plot of  $\ln(R_f)$  vs.  $2(\Phi)$ , which tracks deformation  
136 of elliptical objects in a different and very useful manner (Elliott, 1970), as discussed  
137 below. The program allows the user to save an image of both types of plots.

138 Original and deformed displays can be saved for further study. Data files  
139 containing information about the original or deformed shape, orientation, and position, of  
140 pebbles can be created for statistical analysis or use with commercially available  
141 programs. Snapshots of the display can be captured, and a sequence of such images,  
142 combined with snapshots of the  $R_f$ - $\Phi$  plots, can be used to create animations of

143 deforming pebbles and the corresponding changes in the graphical representation of the  
144 deformation.

145 The program applies the specified linear transformation to simulate deformation  
146 of ellipses in the display area. The program then uses a simple numerical analysis process  
147 to determine the axial ratio of the transformed objects. For each ellipse, it gets a list of  
148 points along the circumference, and then calculates the distance from the center to each  
149 of those points. As it steps through those calculations, it tracks the longest and shortest of  
150 those distances, thus finding the long and short axes. To maximize the accuracy and  
151 efficiency of the axial ratio approximation, the program uses a number of circumference  
152 points close to the number of pixels used to create the ellipse. The analytical equations  
153 for  $\Phi$  and  $R_f$  derived by Ramsay (1967, p. 205-209) and summarized by Lisle (1985, p. 3)  
154 can be used to track deforming ellipses, particularly when their shapes and orientations  
155 are loaded from a file (e.g. Fig 1A). However, when ellipses are traced from an image  
156 (e.g. Fig. 1B), a numerical routine similar to the one we use is needed to determine the  
157 initial shape and orientations of objects. The routine we use mimics what geologist  
158 actually do when they measure naturally deformed elliptical objects.

159

### 160 **3. Demonstrating the $R_f$ - $\Phi$ Method to Students**

161 Figure 2A shows five color-coded groups of ellipses from the display screen with  
162 initial axial ratios equal to 1.2, 1.6, 2.0, 2.4, and 2.8. The long axes of the ellipses are  
163 initially oriented in  $10^\circ$  increments. We created this group of ellipses as a teaching aid  
164 using the Excel Workbook distributed with the program. It is designed to simulate a  
165 group of pebbles with an initially random distribution of long axis orientations, and a  
166 limited range of axial ratios. Figure 2B shows the  $R_f$  vs.  $\Phi$  plot for this ‘undeformed’  
167 array of ellipses. The points are colored the same as the corresponding ellipses, to  
168 facilitate tracking, and they are distributed over the entire range of  $\Phi$  values. Two ellipses  
169 with axial ratios of 2.8, whose long axes are oriented horizontally and vertically are  
170 selected in 2A, and they appear as larger points in 2B to highlight the paths of these  
171 objects during simulated deformation. Also selected in 2A is the large gray unit circle to  
172 help track the value of the “strain ellipse” in the  $R_f$ - $\Phi$  plots. Figure 2C shows the polar  
173 plot of  $\ln(R_f)$  vs.  $2(\Phi)$  for the undeformed array of ellipses. The points are also colored

174 the same as the ellipses in 2A, and the points form a radial distribution around the origin  
175 of the polar plot with more elliptical objects farther from the origin.

176 The ellipses undergo a simulated vertical, pure shear, shortening with strain ratio  
177 of 1.4 (Fig. 2D). The  $R_f$ - $\Phi$  plot (Fig. 2E) shows that the ellipses with initial axial ratios of  
178 1.2 now form a closed loop. The polar plot of  $\ln(R_f)$  vs.  $2(\Phi)$  (Fig. 2F) shows that the  
179 points corresponding to the group of ellipses with an initial axial ratio of 1.2 now lie  
180 entirely to the right of the origin. This pattern reflects the fact that the strain ( $R_s = 1.4$ )  
181 was great enough to transform an initial ellipse ( $R_i = 1.2$ ), whose long axis was  
182 perpendicular to the stretching direction, to a circular object ( $R = 1$ ), and then into an  
183 ellipse whose long axis is parallel to the stretching direction ( $R_f = 1.17$ ). In contrast,  
184 groups of ellipses with initial axial ratios greater than that of the strain ellipse form open  
185 configurations in the  $R_f$ - $\Phi$  plot over the entire range of  $\Phi$  values. This pattern reflects the  
186 fact that elliptical objects, whose axial ratios are greater than the strain ellipse ratio, and  
187 whose long axes are perpendicular to the stretching direction, become less elliptical, but  
188 their long axes do not change orientation during pure shear.

189 Continuing this example, the shortening is increased to a strain ratio of 3:1 (Fig.  
190 2G). At  $R_s = 3$ , points representing all the ellipses form concentric closed loops on the  $R_f$   
191 vs.  $\Phi$  plot (fig. 2H) because  $R_s$  is greater than the maximum initial ellipticity of 2.8. On  
192 the polar plot (fig. 2I), all of the points lie to the right of the origin and together they form  
193 a noticeably elliptical, rather than a circular, distribution.

194 The images in Figure 2 illustrate, in an effective but static fashion, the  
195 relationship between the strained elliptical shapes and the Cartesian and polar  $R_f$ - $\Phi$  plots.  
196 Using the program is much more compelling, however, because the  $R_f$ - $\Phi$  plots are  
197 continuously and instantaneously updated as the ellipses are strained in the display  
198 window, and the user can rapidly perform experiments that visually relate strain to the  
199 plots.

200 To find the  $R_f$  and  $\Phi$  values for individual objects, the user clicks on the point in  
201 the plots and reads the coordinates at the bottom of the window. Thus, it is easy to  
202 determine the values of  $R_{fmax}$  and  $R_{fmin}$  on the  $R_f$  vs.  $\Phi$  plots (Figs. 2E and H), and to use  
203 them to calculate the maximum initial ellipticity,  $R_{imax}$ , and the axial ratio of the strain  
204 ellipse,  $R_s$ , using the equations provided by Ramsay and Huber (1983, p. 77). For

205 example in Figure 2E,  $R_{fmax} = 4.1$  and  $R_{fmin} = 2.1$ . Using these values in the equations for  
206 an open configuration:

207

$$208 \quad (R_{fmax} * R_{fmin})^{1/2} = R_{imax}, \text{ and} \quad (1)$$

209

$$210 \quad (R_{fmax} / R_{fmin})^{1/2} = R_s, \quad (2)$$

211

212 yields  $R_{imax} = 2.9$ , and  $R_s = 1.4$ . In Figure 2H the maximum and minimum values are 8.9  
213 and 1.1. Using these values in the equations for a closed configuration:

214

$$215 \quad (R_{fmax} / R_{fmin})^{1/2} = R_{imax}, \quad (3)$$

216

$$217 \quad (R_{fmax} * R_{fmin})^{1/2} = R_s, \quad (4)$$

218

219 gives  $R_{imax} = 2.8$ , and  $R_s = 3.1$ . Both of these results are in good agreement with the  
220 actual values for strain and maximum initial ellipticity used in the simulations.

221 For an initially random distribution of elliptical objects, finding the  $R_s$  value in the  
222 polar plots is very straightforward. It requires locating the ‘center’ of the points  
223 representing all of the ellipses and clicking on it. The  $\ln(R_f)$  and the  $R_f$  values are given at  
224 the bottom of the plot. One of the inherent challenges of the  $R_f - \Phi$  method is identifying  
225 outliers (pebbles with unusually large initial ellipticity values) on the  $R_f$  vs.  $\Phi$  plot so that  
226 they may be neglected when finding the values of  $R_{fmax}$  and  $R_{fmin}$  to use in Eqs. (1) and  
227 (2) for open configurations or Eqs. (3) and (4) for closed configurations (Ramsay and  
228 Huber, 1983). Using the polar plot circumvents this problem by focusing on the *center* of  
229 the distribution of points, rather than on the maximum and minimum values. Thus, the  
230 correct identification of outliers is much less critical.

231 Figures 2E and H help illustrate an important limitation of the  $R_f - \Phi$  method. It is  
232 only reliable if the undeformed rock contained a large number of objects that possessed  
233 the *greatest* initial axial ratio,  $R_{imax}$ , and the long axes of these objects were not  
234 preferentially oriented. Both criteria must be met to create a robust open (Fig. 2E) or  
235 closed (Fig. 2H) configuration for determining  $R_{fmax}$  and  $R_{fmin}$ . If only a limited number

236 of objects with greatest initial ratio are present, or if the long axes are preferentially  
237 oriented over a limited angular range, the values for  $R_{fmax}$  and  $R_{fmin}$  obtained from  
238 Cartesian  $R_f - \Phi$  plots will be based on pebbles with different initial ellipticity (for  
239 example, purple and yellow ellipses in Fig. 2). This situation will produce an inaccurate  
240 strain estimate. Tracking specific points allows the user to see that after deformation,  
241 objects with maximum initial axial ratios ( $R_i = R_{imax}$ ), and with long axes parallel to the  
242 maximum extension direction, will have the greatest final axial ratio ( $R_{imax} * R_s = R_{fmax}$ ).  
243 In contrast, objects with maximum initial axial ratios (again,  $R_i = R_{imax}$ ), but with long  
244 axes perpendicular to the maximum extension direction will have the minimum final  
245 axial ratio ( $R_{imax}/R_s = R_{fmin}$ ). If the observed values for  $R_{fmax}$  and  $R_{fmin}$  come from unique  
246 outliers, that is to say, pebbles with significantly different initial ratios,  $R_i$ , the strain  
247 estimate will not be reliable.

248

#### 249 **4. Applying the $R_f - \Phi$ Method for Strain Determinations**

250 Figure 3A shows the program window with a background image of a deformed  
251 quartz pebble conglomerate from the Dalton Formation, located in Dalton,  
252 Massachusetts, USA. The photograph was converted into a 700 by 700 pixel image  
253 before being loaded as a background. The blue ellipses were traced from the deformed  
254 pebbles in the editing mode by clicking and dragging with a mouse. An experienced user  
255 can trace fifty pebbles in approximately five minutes, so it is an efficient method for  
256 determining the axial ratio and long axis orientation of deformed pebbles, compared with  
257 measuring individual pebbles with a ruler and protractor. The ellipses generated by the  
258 program rarely conform to the shape of naturally deformed objects. Thus, using the  
259 program to trace pebbles highlights the important, yet commonly overlooked, fact that  
260 naturally deformed objects are not perfect ellipses. As pebble outlines are traced with a  
261 mouse, the corresponding  $R_f$  and  $\Phi$  values are plotted in Cartesian (Fig. 3B) and polar  
262 (Fig. 3C) plots.

263 After the pebbles are traced, the  $R_f - \Phi$  plot (Fig. 3B) can be used to find the  
264 values of  $R_{fmax}$  and  $R_{fmin}$  (as shown in Figures 2E and H). Once these values are obtained,  
265 it is straightforward to calculate the ratio of the strain ellipse,  $R_s$ , and the maximum initial

266 ellipticity,  $R_{\text{imax}}$ , using Eqs. (1) and (2) for open configurations, or Eqs. (3) and (4) for  
267 closed configurations.

268 Another approach, which offers greater insight into the strain state, is to use the  
269 program to find the most likely inverse strain ellipse, and thereby recover the  
270 ‘undeformed’ shapes and distribution of the pebbles. This is most easily done in two  
271 steps, and the process takes advantage of the fact that an arbitrary linear transformation  
272 can be expressed as a pure shear (transformation by a diagonal matrix) followed by a  
273 rotation (transformation by an anti-symmetric matrix). These steps are reversed for an  
274 inverse transformation (inverse strain).

275 Figure 3D shows the pebble outlines after a counter-clockwise rigid rotation of  $9^\circ$ ,  
276 a value found by trial and error and chosen because it creates a symmetric distribution of  
277 points about the  $\Phi = 0$  axis in the  $R_f$  vs.  $\Phi$  plot (Fig. 3E), and the  $2\Phi = 0$  axis in the polar  
278  $\ln(R_f)$  vs.  $2\Phi$  plot (Fig. 3F). The second step is to apply horizontal shortening by pure  
279 shear, and create the most dispersed possible distribution of points over the entire range  
280 of  $\Phi$  values (Fig. 3H). A dispersed pattern of points, as seen in Figure 3H, simulates an  
281 initially random distribution of long axis orientations. An equivalent approach is to create  
282 a radial distribution of points centered at the origin of the polar plot of  $\ln(R_f)$  vs.  $2\Phi$  to  
283 simulate an initially random distribution of long axis orientations (Fig. 3I).

284 The axial ratio of the inverse strain ellipse determined by the inverse graphical  
285 method is 3.7. This strain estimate is similar to the value of  $R_s = 3.67$  as calculated using  
286 Eq. (4) with values of  $R_{\text{fmin}} = 1.95$  and  $R_{\text{fmax}} = 6.93$ , as measured from Figure 3B. Likely  
287 outliers were neglected when selecting the  $R_{\text{fmin}}$  and  $R_{\text{fmax}}$  values. By comparison, field  
288 measurements at the outcrop using a ruler and protractor gave a strain estimate of 3.75  
289 using the standard  $R_f - \Phi$  method (Ramsay and Huber, 1983). As expected, the agreement  
290 between the three approaches is excellent. The inverse graphical strain method, however,  
291 has the advantage of allowing visual inspection of the starting configuration of the  
292 pebbles. The value of direct field measurements cannot be overstated, and should always  
293 be the highest priority. However, it is not always possible to measure objects on  
294 inaccessible surfaces in the field, and our program is well suited for working with  
295 photographs of hard-to-reach outcrop surfaces, as well as photographs of slabbed hand  
296 samples and thin sections.

297           The graphical approach for finding inverse strain described above is valuable  
298 because the initial shapes of the pebbles, and their spatial distribution, can be easily  
299 portrayed (Fig. 3G). This functionality makes it possible for researchers and students to  
300 assess quickly and *visually* the plausibility of strain determinations using the  $R_f\text{-}\Phi$   
301 method. Furthermore, although the examples presented here assume no area change, the  
302 program can incorporate area change if such data are available. Because the program runs  
303 graphical experiments very quickly, it is also practical to explore the effects of varying  
304 area change if the direction of area loss can be deduced from stylolites or pebble  
305 indentations (Onasch, 1984).

306

## 307 **5. Conclusions**

308           Although we do not provide new insight into the theory behind the  $R_f\text{-}\Phi$  method,  
309 we believe our program advances strain studies for both educational and research  
310 purposes. Our main goal in creating this program was to provide a *visual* link between  
311 deforming elliptical objects and corresponding  $R_f\text{-}\Phi$  plots, and we endeavored to do this  
312 with an easy-to-learn application so attention can be focused on strain rather than  
313 learning how to use the program. Instructors can use the main display window to  
314 demonstrate fundamental principles of strain, such as the difference between coaxial and  
315 non-coaxial strain, and the relationship between strain and deformation fabric. The  
316 program also allows students to see how deformation of a group of ellipses is manifested  
317 in  $R_f\text{-}\Phi$  plots, and how strain can be estimated from such a population. The digitizing  
318 capabilities of the program make it possible for students to quickly generate  $R_f\text{-}\Phi$  plots  
319 from photographs of deformed conglomerate or oolitic limestone, making it more  
320 practical to assign such problems.

321           The program generates both the familiar Cartesian  $R_f\text{-}\Phi$  plots and polar plots of  
322  $\ln(R_f)$  vs.  $2(\Phi)$ , as suggested by Elliott (1970). Direct comparison of the two kinds of  
323 plots highlights the advantages and disadvantages of each. Although the polar plots are  
324 somewhat more difficult to relate to the shape and orientation of elliptical objects, the  
325 program demonstrates that the initial array of points representing ellipses is displaced  
326 with only moderate distortion in the  $\ln(R_f)$  vs.  $2(\Phi)$  plots (Figs. 2 & 3). Recognition and

327 elimination of statistical outliers is also much less critical when using polar than  
328 Cartesian plots.

329         Researchers can use the program to digitize deformed objects from imported  
330 photographs of outcrops, slabbed hand samples, and thin sections. Strain can be  
331 determined directly from the  $R_f$ - $\Phi$  plots using the standard equations, or by retro-  
332 deformation of the ellipses, as described above. The advantage of the latter approach is  
333 that it gives a view of the initial shape and distribution of objects, and thereby permits a  
334 visual assessment of the validity of the strain estimate. The program can model area  
335 change during deformation, and graphical experiments can be performed rapidly, so it is  
336 feasible to explore the effects of area change when evidence for pressure solution exists  
337 (Onasch, 1984). The program does not provide automated strain determinations or testing  
338 of initially random distribution of ellipses. It is easy, however, to export data files with  
339 information about the position, axial ratio, and long axis orientation of ellipses. Such files  
340 can then be used for statistical and graphical analysis to test rigorously strain models, and  
341 the assumptions on which they are based. Data files generated by our program can also be  
342 imported into commercially available programs that perform symmetry and distribution  
343 tests on deformed objects.

344         The program, which we call GeoShear, along with a photograph of a deformed  
345 conglomerate, files of synthetic ellipses, and an Excel workbook that serves as a template  
346 for creating new files can be downloaded at:

347 <http://www.williams.edu/Geoscience/facultypages/Paul/currentresearch.html>.

348

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381

382 **Figure and Table Captions**

383

384 **Figure 1.** Screen captures of the entire program window. **A.** Individual ellipses can be  
385 drawn in editing mode with a wide variety of colors. **B.** Photographs (up to 700 by 700  
386 pixels) of deformed objects, such as these quartz pebbles, are imported in the display  
387 window for tracing ellipses as shown in blue.

388

389 **Figure 2.** Screen captures of the display window and Cartesian and polar  $R_f$ - $\Phi$  plots. **A.**  
390 Undeformed (axial ratio of strain ellipse,  $R_s = 1$ ) array of color-coded ellipses. **B.** Plot of  
391  $R_f$  vs.  $\Phi$  for the unstrained configuration of ellipses shown in A. **C.** Polar plot of  $\ln(R_f)$   
392 vs.  $2\Phi$  for the unstrained configuration of ellipses shown in A. **D.** Ellipses shown in A  
393 after vertical shortening by pure shear such that the axial ratio of the strain ellipse,  $R_s$ , is  
394 equal to 1.4. **E.** Plot of  $R_f$  vs.  $\Phi$  for the ellipses shown in D. **F.** Polar plot of  $\ln(R_f)$  vs.  $2\Phi$   
395 for the ellipses shown in D. **G.** Ellipses shown in A after vertical shortening by pure shear  
396 such that the axial ratio of the strain ellipse,  $R_s$ , is equal to 3. **H.** Plot of  $R_f$  vs.  $\Phi$  for the  
397 ellipses shown in G. **I.** Polar plot of  $\ln(R_f)$  vs.  $2\Phi$  for the ellipses shown in G. Note that  
398 the display screens in D and G were rescaled to show all the pebbles after deformation.  
399  $R_{f_{\max}}$  and  $R_{f_{\min}}$  in E and H are the maximum and minimum axial ratios used to find the  
400 values of the strain ellipse,  $R_s$ , and maximum initial axial ratio,  $R_{i_{\max}}$ , in Eqs. (1) through  
401 (4).

402

403 **Figure 3.** Screen captures of the entire program window and the Cartesian and polar  $R_f$ -  
404  $\Phi$  plots. **A.** Photograph (700x700 pixels) of deformed quartz-pebble conglomerate  
405 imported as background image. **B.** Plot of  $R_f$  vs.  $\Phi$  of the deformed pebbles in A. **C.**  
406 Polar plot of  $\ln(R_f)$  vs.  $2\Phi$  of the deformed pebbles in A. **D.** Pebble outlines after  $9^\circ$  of  
407 counter-clockwise rigid rotation. **E.** Plot of  $R_f$  vs.  $\Phi$  of the pebbles after  $9^\circ$  of counter-  
408 clockwise rigid rotation. **F.** Polar plot of  $\ln(R_f)$  vs.  $2\Phi$  of the pebbles after  $9^\circ$  of counter-  
409 clockwise rigid rotation. **G.** Pebble outlines after horizontal shortening by pure shear such  
410 that the axial ratio of the inverse strain ellipse is equal to 3.7. Note that the display screen  
411 in G was rescaled to show all the pebbles after deformation. **H.** Plot of  $R_f$  vs.  $\Phi$  of the

412 pebbles after horizontal shortening by pure shear such that the axial ratio of the inverse  
413 strain ellipse is equal to 3.7. The inverse strain value was determined by trial and error to  
414 create the most dispersed distribution of points over the entire range of  $\Phi$  values to  
415 simulate an initially random distribution of long axis orientations. **I.** Polar plot of  $\ln(R_f)$   
416 vs.  $2\Phi$  of the pebbles after horizontal shortening by pure shear such that the axial ratio of  
417 the inverse strain ellipse is equal to 3.7. The inverse strain value was determined by trial  
418 and error to create a radial distribution of points centered at the origin to simulate an  
419 initially random distribution of long axis orientations.

420

421 **Table 1.** Abbreviations used in text and figures.

422

423 **Table 2.** Abbreviated explanation of GeoShear command buttons to illustrate available  
424 functions.

Figure 1

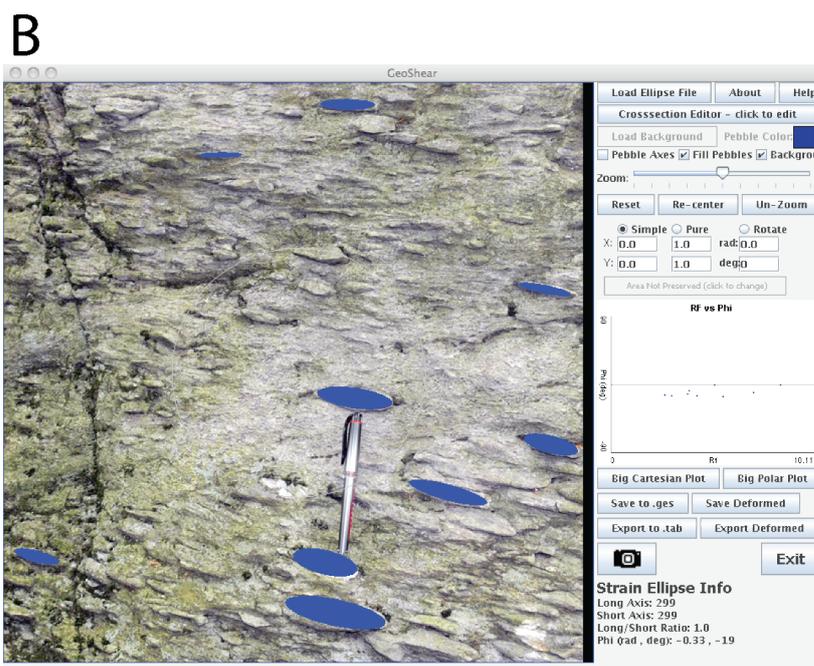
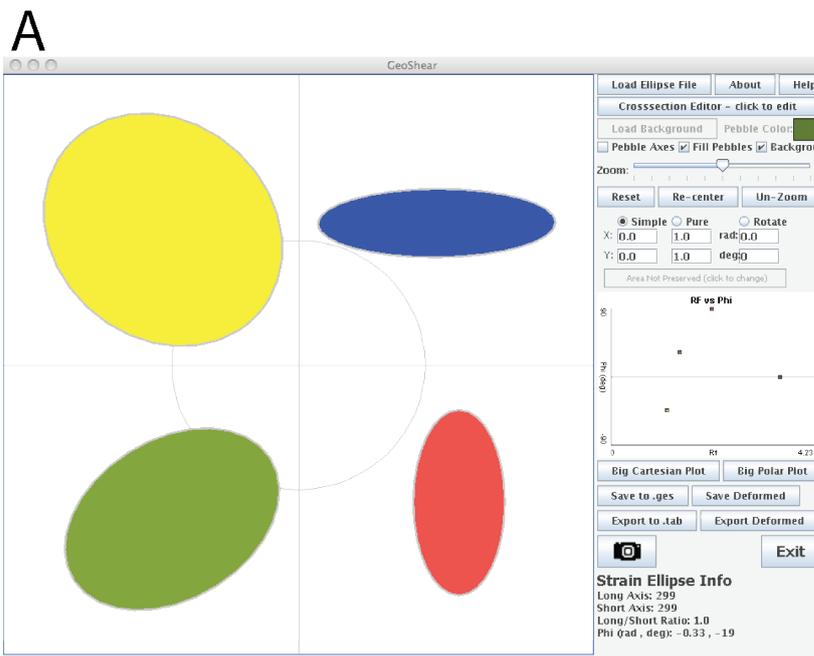


Figure 1.

Figure 2

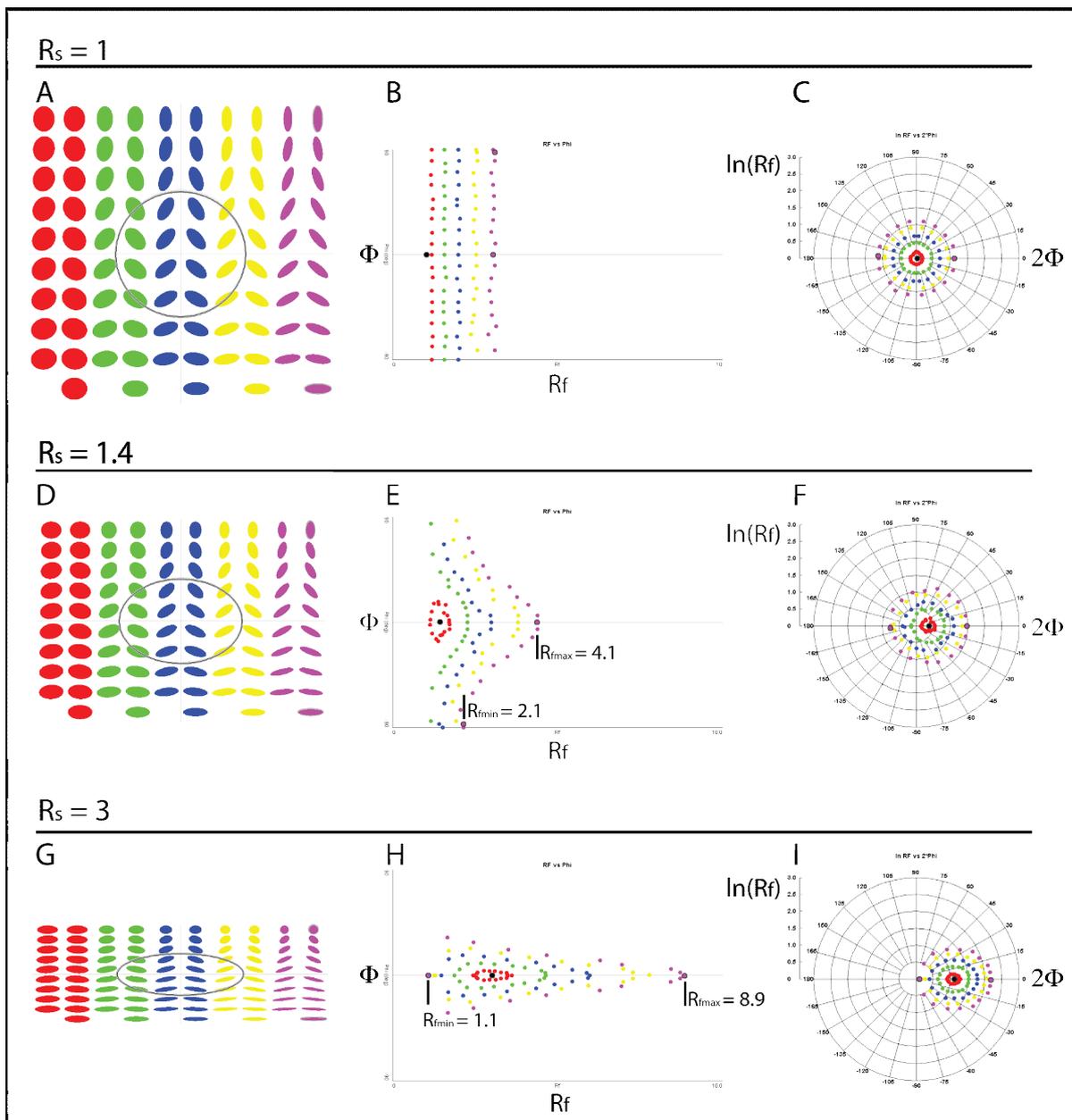


Figure 2.

Figure 3

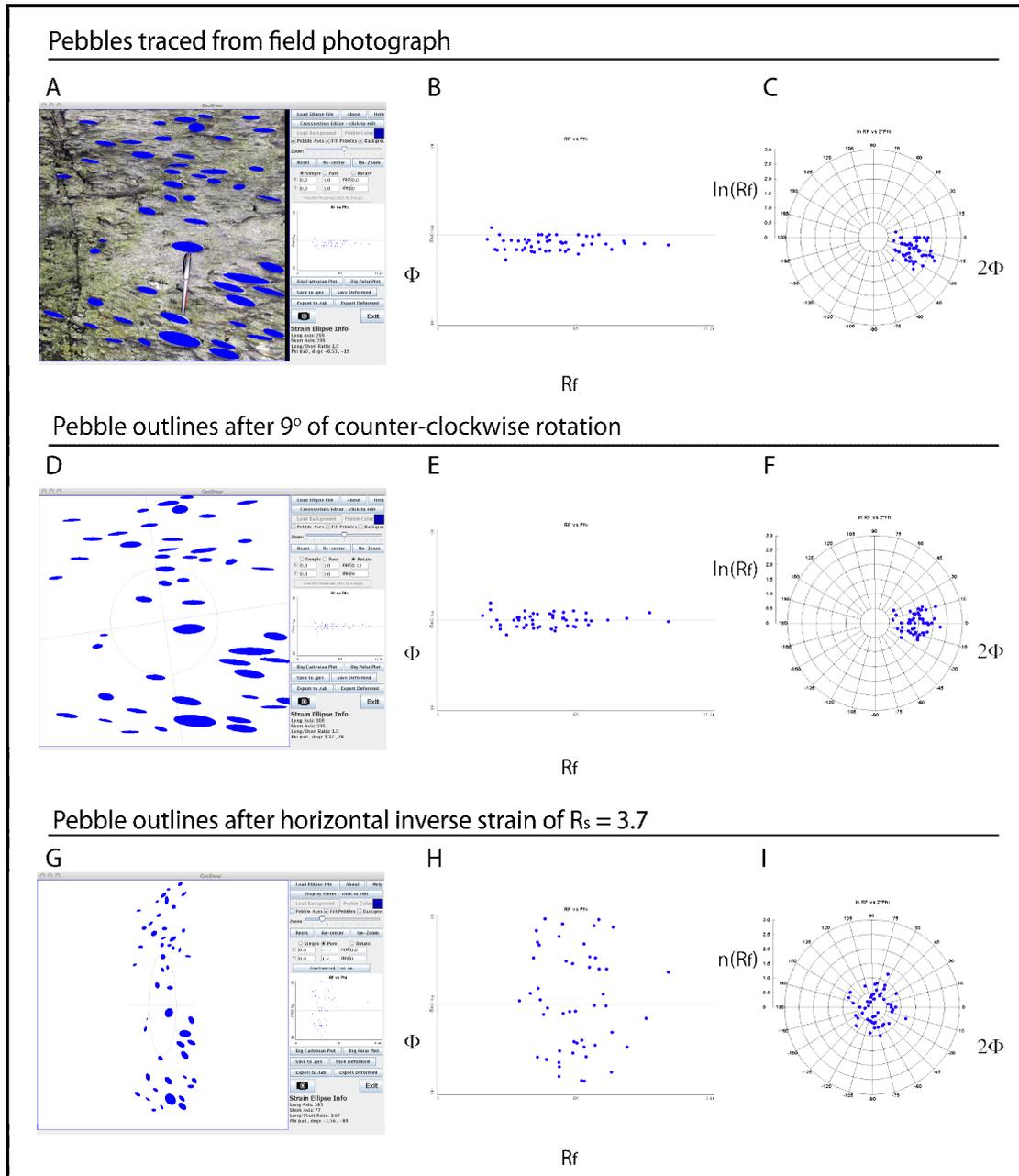


Figure 3.

Table 1. Abbreviations used in text and figures.

$\Phi$	The angle between an arbitrary reference line and the long axis of an ellipse. Range: $-90^\circ$ to $90^\circ$ .
$R_f$	The final axial ratio (long axis/short axis) of any arbitrary elliptical object.
$R_{fmax}$	The maximum final axial ratio of all the elliptical objects.
$R_{fmin}$	The minimum final axial ratio of all the elliptical objects.
$R_i$	The initial axial ratio of any arbitrary elliptical object.
$R_{imax}$	The maximum initial axial ratio of all the elliptical objects.
$R_s$	The axial ratio of the strain ellipse (long axis/short axis).

Table 2. Abbreviated explanation of GeoShear command buttons to illustrate available functions.

Command	Function
<b>Load Ellipse File</b>	Loads an existing ellipse data file that contains shape, orientation position, and color information.
<b>Help</b>	Brings up the help window.
<b>About</b>	Displays information about this program.
<b>Display Editor</b>	Allows you to draw elliptical objects with the mouse, load a background image, and trace ellipses from a background image.
<b>Load Background</b>	Loads an image background for tracing elliptical objects.
<b>Pebble Color</b>	Double-click on the box to change the color. Change the color of all selected pebbles by ALT-clicking on the color box.
<b>Pebble Axes</b>	Toggles between showing and hiding long and short axes of elliptical objects.
<b>Fill Pebbles</b>	Toggles between filled and outlined pebbles.
<b>Background</b>	Toggles between showing and hiding the loaded background.
<b>Zoom</b>	Change the magnification of the cross section from 20% to 500%.
<b>Reset</b>	Removes all deformation, positioning, and magnification changes.
<b>Re-Center Display</b>	Re-centers the display but preserves deformation. The display may be moved off center by holding down the ALT key and dragging in the display area.
<b>Un-Zoom</b>	Returns display to the default magnification of 100%.
<b>Simple shear</b>	Constrains deformation to be by simple shear. Click and drag in the display to deform elliptical objects. You can also type in exact values for simple shear in the horizontal and/or vertical directions in the rectangles below the radio button.
<b>Pure shear</b>	Constrains deformation to be by pure shear. Click and drag in the display to deform elliptical objects. You can also type in exact values for extension in the horizontal and vertical directions in the rectangles below the radio button.
<b>Area Not Preserved (click to change)</b>	Toggles between three options. The first allows area to change during pure shear. <b>Area Preserved, Y Ind. Var.</b> lets you specify the vertical extension and adjusts horizontal extension to preserve area. <b>Area Preserved, X Ind. Var.</b> lets you specify the horizontal extension and adjusts vertical extension to preserve area.
<b>Lock in current deformation</b>	Used when switching from simple shear, pure shear, or rotation, to another deformation mode. Resets frame of reference.
<b>Rotate</b>	Constrains deformation to be by rotation. Click and drag in the display to rotate objects. You can also type in exact rotation angles as either degree or radians.
<b>Big Cartesian Plot</b>	A larger Cartesian $R_f$ vs. $\Phi$ plot opens up in a new, re-sizeable window if this button is pressed.
<b>Big Polar Plot</b>	A polar plot of $\ln(R_f)$ vs. $2(\Phi)$ opens up in a new, re-sizeable window if this button is pressed.
<b>Save to ges.</b>	Save the starting display to a file. The file retains the background and all the elliptical objects, but not subsequent deformation or other display changes.
<b>Save Deformed</b>	Save a deformed display, but not the original shapes.
<b>Export to tab.</b>	Saves the undeformed pebble information to a tab-delimited file.
<b>Export Deformed</b>	Saves the deformed pebble information to a tab-delimited file.
<b>Camera</b>	Take a snapshot of the current display. It will capture whatever is in the display area as a PNG, BMP, or JPG image.
<b>Exit</b>	Exits the program without saving anything.
<b>Strain Ellipse Info</b>	This area displays information about the strain ellipse (the grey circle in the center that you can see when no background is displayed).

