Exercise 2

These computational exercises should be completed by **January 8** at **11:59AM**. Solutions should be turned in through the course website.

1. **Cube Roots**

   Modify the square root program `my_sqrt.py` to calculate the cube root of a number. (Note: The update step requires some thought).

2. **Square Root by Taylor Series**

   The function $\sqrt{1+x}$ has the following Taylor series

$$\sqrt{1+x} = \sum_{j=0}^{\infty} a_j x^j \tag{1}$$

   where $a_0 = 1$, $a_1 = \frac{1}{2}$, and

$$a_{j+1} = -\left(\frac{2j-1}{2j+2}\right) a_j. \tag{2}$$

   Modify the exponential function `exp3` to evaluate the square root of $y = 1 + x$ using this method. What happens when $x = 1$? What about $x = -1$?

   The problem, of course, is that the Taylor series does not converge for $|x| \geq 1$. One way to fix this is to use the identity

$$\sqrt{y} = n\sqrt{1 + \frac{y-n^2}{n^2}} \tag{3}$$

   with an integer n such that $x = (y-n^2)/n^2$ is sufficiently small. Use this method to calculate $\sqrt{2}$, $\sqrt{10}$, and $\sqrt{15}$ using the Taylor series. How do the number of iterations compare using this method and the

3. **Exponential Function by Scaling and Squaring**

   While the Taylor series of the exponential always converges, it still does a rather poor job of evaluating e^x when x is large. Use the identity $e^x = (e^{x/n})^n$ to reduce the problem to evaluating e^y where $y = x/n$ is in the range $[0, 1]$. Write a program that finds the integer n, evaluates $e^{x/n}$ using the Taylor expansion method, and finally calculates e^x by the product method. An optimal implementation uses $n = 2^m$, and m squaring operations.

4. **Challenges**

   - Write a code that evaluates the n-th root of any number by the Babylonian method.

- Write a code to evaluate the square root of any number by the Taylor series method.
- Use the series expansions of $\sin(x)$ and $\cos(x)$ to evaluate several values between 0 and 10.
- Look up the series expansion of the inverse tangent. Use it to evaluate $\arctan(1) = \pi/4$.